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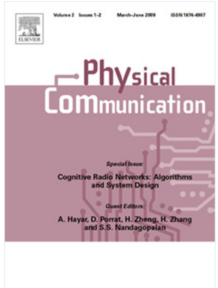

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PII: S1874-4907(21)00184-1
DOI: https://doi.org/10.1016/j.phycom.2021.101447
Reference: PHYCOM 101447

To appear in: *Physical Communication*

Received date : 13 April 2021
Revised date : 15 July 2021
Accepted date : 24 August 2021

Please cite this article as: M. Li, S. Huang, L. Tian et al., Error rate performance of NOMA system with full-duplex cooperative relaying, *Physical Communication* (2021), doi: https://doi.org/10.1016/j.phycom.2021.101447.

Error Rate Performance of NOMA System with Full-duplex Cooperative Relaying

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Abstract

This paper studies a FD cooperative non-orthogonal multiple access (NOMA) system in terms of pairwise error probability (PEP) while considering imperfect successive interference cancellation (ipSIC). Exact closed-form expressions of the PEP performance for the system users are derived. The PEP expressions are derived under both ideal condition and in the presence of self-interference due to the FD relaying. Moreover, we derive asymptotic PEP expressions to evaluate an effective diversity order for the considered system. Through the derived (asymptotic) expressions and simulation results, we demonstrate that self-interference due to FD relaying has a great influence on the performance of the full-deplex relay.

Keywords: Non-orthogonal multiple access, cooperative relay, pairwise error probability, diversity gain

1. Introduction

Non-orthogonal multiple access (NOMA) is one of the key-enablers of fifth-generation (5G) mobile communication systems. NOMA can effectively improve spectrum efficiency by using power domain to serve multiple users on the same resource block (in time domain, frequency domain or code domain) [1] [2]. The basic feature behind NOMA is to equip a receiver with some prior infor-

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mation of the expected messages from other transmitters for successive interference cancellation. Such prior information can be utilized to further improve the system performance.

Cooperative diversity, mostly used in infrastructure-less networks, emerged in the past decade as a promising approach to increase the spectral and power efficiencies, broaden network coverage, and reduce the outage probability [3]. Cooperative relaying is introduced to NOMA to obtain spatial diversity gain [4]. In cooperative NOMA, a user with good channel condition can be regarded as a relay to forward signal to another user with a relatively worse channel condition [5]. In doing so, the challenged (or far-away) user can achieve a spatial diversity gain. There mainly exist two kind of cooperative methods: half-duplex (HD) mode and full-duplex (FD) mode.

In [6], the authors investigate a relay selection scheme in half-duplex (HD)-based Decode-and-Forward (DF) relaying to enhance the outage performance. In [7], the authors propose cooperative NOMA with HD relay selection. To further improve the bandwidth efficiency, Full-duplex (FD) relay technology is a promising solution which can simultaneously receive and transmit signals in the same frequency band although it suffers from residual loop self-interference (SI) [8]. The authors of [9] investigate the outage performance of secondary users in underlay cognitive radio–based nonorthogonal multiple access networks, in which the near user is used as the full-duplex decode-and-forward (DF) relay for delivering the message to the far-user. Besides, an optimal location for the FD relay node is investigated for the purpose of minimizing the outage probability of far-user. In [10] [11], the authors investigate two-stage transmission scheme and the ergodic sum rate and outage probability of a cooperative NOMA system have been evaluated. In [12], the authors study the impact of relay selection on the performance of FD cooperative NOMA. In NOMA, the receivers use successive interference cancellation (SIC) to avoid co-channel interference and obtain their desired signals. Most of the current research assumed SIC with perfect inteference cancellation. However, this assumption is impractical due to the inaccurate power allocation and imperfect channel decoding [13]. Several results have been reported on the effect of imperfect SIC (ipSIC) on the performance of NOMA systems [14]. However, all existing works focus on evaluating the performance of NOMA systems in terms of outage probability, ergodic sum rate, individual sum rate and spectral efficiency.

Error rate performance can offer a useful insight into the quality of service of all users [15, 16].

However, accurate bit error rate (BER) analysis of NOMA systems with ipSIC is intractable, which motivates the evaluation of the PEP as an upper bound of BER [17]. As far as the authors known that only the references [18–21] studied the system performance of NOMA in terms of PEP. In literature [18], the authors derived the closed PEP expressions of non-cooperative NOMA system under Nakagami-m channel, and analyzed the effective diversity gain of the system. In literature [19], the authors studied the impact of residual hardware impairment on the PEP of non-orthogonal multiple access by taking into account the effect of imperfect successive interference cancellation (ipSIC). In literature [20], the authors derived the PEP asymptotic expression of collaborative NOMA system with RHI and analyzed the diversity gain. In literature [21], the authors studied the PEP performance of secondary users under CR-NOMA network and considered the problem of optimal relay selection. However, the work in existing literature did not address the upper bound of error rate performance analysis in NOMA system with full-duplex cooperative relaying by taking into account the ipSIC implementation.

The rest of this paper is organized as follows: Section 2 shows the considered system model in the manuscript. Section 3 derives an exact closed-form PEP expression characterizing the performance of FD cooperative NOMA with ipSIC. In Section 4, an asymptotic PEP expression is given to investigate the achievable diversity gains of the NOMA users under both ideal and non-ideal cases. Section 5 gives numerical results to verify analytical expressions. Finally, Section 6 draws some conclusions.

2. System model

Different from the existing AF cooperative NOMA system, a downlink FD cooperative NOMA system is depicted in Figure 1, which consists of a base station (BS) and two users, U_1 and U_2, which are located in near and far away from the BS, respectively. Due to physical obstacles or heavy shadowing, we asume that there is no direct link from the BS to U_2 [4]. Therefore, to aid with the data delivery from the BS to U_2, U_1 acts as a full-duplex relay.

The channels from the BS to U_1, and U_1 to U_2 are denoted by h_1 and h_2, respectively. Both channels are modeled as independent and identically distributed (i.i.d.) Rayleigh flat fading channels, i.e., $h_i \sim \mathcal{CN}\left(0, \delta_i^2\right)$, $i \in \{1, 2\}$, where $\mathcal{CN}\left(0, \delta_i^2\right)$ denotes the distribution of the complex

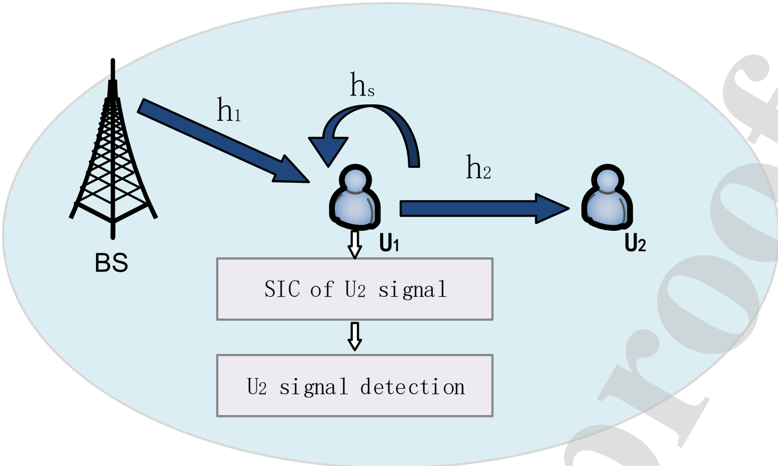

Figure 1: The downlink NOMA system model

Gaussian random variable with zero mean and variance δ_i^2.

Let $x_1(t)$ and $x_2(t)$ be the messages intended from the BS to U_1 and U_2, respectively. The BS superimposes both messages as NOMA signal as follows,

$$x(t) = \sqrt{\alpha_1 p} x_1\left(t\right) + \sqrt{\alpha_2 p} x_2\left(t\right), \tag{1}$$

where p is the BS transmit power, α_1 and α_2 are the power allocation parameters for U_1 and U_2, respectively, with $\alpha_1 + \alpha_2 = 1$, $\alpha_1 < \alpha_2$, and $E[|x\left(t\right)|^2] = 1$. When the BS broadcasts NOMA signals over the network, the relay user U_1 receives $x(t)$, and attempts to decode the target message $x_2\left(t\right)$ using successive interference cancellation (SIC). Specially, we consider the practical case, i.e., perfect SIC can not be realized, so U_1 can not fully decode the signal of x_2, we denote the estimated signal of $x_2(t)$ at U_1 by $x_2'(t)$. Correspondingly, U_1 forwards $s\left(t\right) = x_2'\left(t-\tau\right)$, where τ is the processing delay at U_1 due to decoding $x(t)$. To this end, two signals will be conveyed to U_1 simultaneously, namely the NOMA composite signal from the BS, and the self-interference signal from the U_1's full-duplex mode. The received signal at U_1 can be expressed as

$$y_1\left(t\right) = \sqrt{\alpha_1 p} h_1\left(t\right) x_1\left(t\right) + \underbrace{\sqrt{\alpha_2 p} h_1\left(t\right)\Delta_2}_{\text{ipSIC}} + \underbrace{h_s\left(t\right)\sqrt{k p_1} s\left(t\right)}_{\text{Self-interference}} + \underbrace{n_1\left(t\right)}_{\text{Noise}}, \tag{2}$$

where p_1 is the transmit power at U_1, $k \in [0,1]$ denotes the self-interference factor, $h_s \sim \mathcal{CN}\left(0, \delta_s^2\right)$

is the self-interference channel; $n_1 \sim \mathcal{CN}(0, \sigma_n^2)$ is a complex Gaussian noise; and $\Delta_2 = x_2 - x_2'$. The received signal at U_2 is given by

$$y_2(t) = h_2(t)\sqrt{p_1}s(t) + n_2(t), \tag{3}$$

where $n_2 \sim \mathcal{CN}(0, \sigma_n^2)$ is a complex Gaussian noise at U_2.

3. PEP Analysis

3.1. PEP Analysis for U_1

The PEP is defined as the probability of detecting message $\hat{x}$ given that message x was transmitted. For U_1, it is necessary to extract signal x_2 using SIC before decoding its own signal x_1. Therefore, the PEP for U_1 can be expressed as

$$P(x_1, \hat{x}_1) = P(x_1, \hat{x}_1|\Omega)\Pr(\Omega) + P(x_1, \hat{x}_1|\bar{\Omega})\Pr(\bar{\Omega}), \tag{4}$$

where $\Omega\left(\bar{\Omega}\right)$ denotes U_1 can decode the signal x_2 successfully (unsuccessfully). We can obtain the conditional PEP as follows:

$$P(x_1, \hat{x}_1|\,\Omega, |h_1|) = \Pr(|y_1 - \sqrt{\alpha_1 p}h_1\hat{x}_1|^2 \le |y_1 - \sqrt{\alpha_1 p}h_1 x_1|^2), \hat{x}_1 \ne x_1. \tag{5}$$

Substituting (2) into (5) with some further modifications, we obtain

$$P(x_1, \hat{x}_1|\Omega, |h_1|) = \Pr\left(2\Re\{h_1\Delta_1 n_1^*\} \le -|h_1|^2\sqrt{\alpha_1 p}|\Delta_1|^2 - 2|h_1|^2\Re\{\Delta_1\sqrt{\alpha_2 p}\Delta_2^*\} - 2\Re\{h_1\Delta_1\sqrt{kp_1}h_s s^*\}\right), \tag{6}$$

where $\Delta_1 = x_1 - \hat{x}_1$ and $2\Re\{h_1\Delta_1 n_1^*\} \sim \mathcal{N}(0, 2\sigma_n^2|h_1|^2|\Delta_1|^2)$, $\Re\{Z\}$ represents the real part of the complex number Z.

Also following [18], (6) can be further expressed as

$$P(x_1, \hat{x}_1|\Omega, |h_1|) = Q(\frac{|h_1|\psi_1 + \sqrt{k}|h_s|\psi_s}{\varsigma_1}), \tag{7}$$

where, $\psi_1 = \sqrt{\alpha_1 p}|\Delta_1|^2 + 2\Re\{\Delta_1\sqrt{\alpha_2 p}\Delta_2^*\}$, $\psi_s = 2\Re\{\Delta_1\sqrt{p_1}s^*\}$, $\varsigma_1 = \sqrt{2}\sigma_n|\Delta_1|$, and $Q(\cdot)$ is the Gaussian Q-function [18]. In this paper, we consider that the channel experienced by self-interference in full-duplex relay forwarding is Rayleigh fading, i.e.,$f_{|h_x|}(\omega) = \frac{\omega_x}{\delta_x^2}\exp\left(-\frac{\omega_x^2}{2\delta_x^2}\right)$, where $\delta_x^2 = E[|h_x|^2]$. According to the principle of NOMA, the SINR of U_1 to detect x_2 from (2) is

$$\gamma_{1,2} = \frac{\alpha_2 p|h_1|^2}{\alpha_1 p|h_1|^2 + kp_1|h_s|^2 + \sigma_n^2}. \tag{8}$$

Let γ_2 is the threshold for U_1 detect x_2, using [22, eq. 365.3.462.5] we obtain the probability of successful decoding of x_2 at U_1 as

$$\Pr(\Omega) = \Pr(\gamma_{1,2} > \gamma_2) = \frac{2\tau_2\upsilon - \sqrt{2\pi\upsilon}\tau_1}{\tau_2\delta_s^2}\exp\left(\frac{\tau_1^2 - \gamma_2^2\sigma_n^4\delta_s^2\tau_2}{2\delta_1^2\delta_s^2p^2\beta^2\tau_2}\right)Q\left(\frac{\sqrt{\upsilon}\gamma_2^2kp_1\sigma_n^2}{(\delta_1 p\beta)^2}\right), \tag{9}$$

where $\beta = \alpha_2 - \gamma_2\alpha_1$, $\tau_1 = \delta_s^2\gamma_2^2kp_1\sigma_n^2$, $\tau_2 = (\delta_s\gamma_2kp_1)^2 + (\delta_1 p\beta)^2$, and $\upsilon = \frac{(\delta_1\delta_s p\beta)^2}{(\delta_s\gamma_2kp_1)^2 + (\delta_1 p\beta)^2}$.

Note that, when the self-interference is completely eliminated (i.e., $k = 0$), (9) can be reduced to

$$\Pr(\Omega) = \exp\left(-\frac{\gamma_2^2\sigma_n^4}{2\delta_1^2p^2\beta^2}\right). \tag{10}$$

To further simplify (7), we consider two scenarios: ideal and non-ideal self-interference cancellation.

3.1.1. Ideal self-interference cancellation

For $k = 0$, the PEP of U_1 under successful decoding of x_2 is

$$P_i(x_1, \hat{x}_1|\Omega) = \int_0^\infty \frac{\omega_1}{\delta_1^2}\exp\left(-\frac{\omega_1^2}{2\delta_1^2}\right)Q\left(\frac{\omega_1\psi_1}{\varsigma_1}\right)d\omega_1. \tag{11}$$

Using [22, eqs. (646.6.287.2-887.8.250.1)], we obtain

$$P_i(x_1, \hat{x}_1|\Omega) = \frac{1}{2}\left(1 - \frac{\psi_1\delta_1}{\sqrt{\varsigma_1^2 + \psi_1^2\delta_1^2}}\right). \tag{12}$$

We assume that the error probability for U_1 to decode x_1 is no more than 1/2, here we consider the worst case. After substituting (10) and (12) into (4), the PEP of U_1 can be rewritten as

$$P_i(x_1, \hat{x}_1) = \frac{1}{2}(1 - \frac{\psi_1\delta_1}{\sqrt{\varsigma_1^2 + \psi_1^2\delta_1^2}}) \exp(-\frac{\gamma_2^2\sigma_n^4}{2\delta_1^2 p^2\beta^2}) + \frac{1}{2}(1 - \exp(-\frac{\gamma_2^2\sigma_n^4}{2\delta_1^2 p^2\beta^2})). \tag{13}$$

3.1.2. Non-ideal self-interference cancellation

For an arbitrary k, the PEP of U_1 conditioned on successfully decoding x_2 is derived by averaging (7) over $|h_1|$ and $|h_s|$. Using the Chernoff bound for the Q-function, $Q(x) \leq e^{-\frac{x^2}{2}}$, and after some mathematical manipulations, we obtain

$$P_{ni}(x_1, \hat{x}_1|\Omega) \leq \frac{1}{\delta_s^2\delta_1^2}\int_0^\infty \omega_s \exp(-\omega_s^2\frac{\varsigma_1^2 + k\psi_s^2\delta_s^2}{2\delta_s^2\varsigma_1^2})\int_0^\infty \omega_1 \exp(-\omega_1^2\frac{\psi_1^2\delta_1^2 + \varsigma_1^2}{2\varsigma_1^2\delta_1^2} - \omega_1\frac{\sqrt{k}\psi_1\psi_s\omega_s}{\varsigma_1^2})d\omega_1 d\omega_s. \tag{14}$$

Using [22, eq. 365.3.462.5], [22, eq. 337.3.326.2.[10]]. and [22, eq. 647.6.292], (14) is further simplified as

$$P_{ni}(x_1, \hat{x}_1|\Omega) \leq \frac{\varsigma_1^2}{\delta_s^2(\psi_1^2\delta_1^2 + 2\varsigma_1^2)}I_1 - \frac{\varphi_1}{\delta_s^2\delta_1^2}I_2, \tag{15}$$

where I_1 and I_2 can be obtained as

$$I_1 = \frac{\varsigma_1^2\delta_s^2}{\varsigma_1^2 + k\psi_s^2\delta_s^2}, \tag{16a}$$

$$I_2 = \frac{1}{2\sqrt{\pi}\varphi_s^3}\left[\frac{\arctan\mu_1}{\mu_1^3} - \frac{1}{\mu_1^2(\mu_1^2+1)}\right], \tag{16b}$$

where $\mu_1 = \sqrt{\frac{k\psi_s^2\delta_s^2+\varsigma_1^2}{2\delta_s^2\varsigma_1^2\varphi_s^2} - 1}$, $\varphi_1 = \frac{\sqrt{k}\psi_1\psi_s\delta_1^2}{2(\psi_1^2\delta_1^2+\varsigma_1^2)}\sqrt{\frac{2\pi\varsigma_1^2\delta_1^2}{\psi_1^2\delta_1^2+\varsigma_1^2}}$ and $\varphi_2 = \frac{\sqrt{k}\psi_1\psi_s\delta_1}{\sqrt{2}\varsigma_1\sqrt{\psi_1^2\delta_1^2+\varsigma_1^2}}$. The PEP of U_1 in the non-ideal self-interference scenario can be also obtained by substituting (9) and (15) into (4).

3.2. PEP Analysis for U_2

As for the relay work, U_1 forwards x_2 (i.e.,the successfully decoded message of U_2). Yet, the PEP of U_2 can occur due to the propagation error of the wireless channel. Therefore, when U_1

correctly decode message x_2, the PEP of U_2 can be expressed as

$$P(x_2, \hat{x}_2||h_2|) = \Pr(2\Re\{h_2\Delta_2' n_2^*\} \leq -|h_2|^2|\Delta_2'|^2\sqrt{p_1}), \tag{17}$$

where $\Delta_2' = x_2(t-\tau) - \hat{x}_2(t-\tau)$. According to [18] , (17) can be calculated as

$$P(x_2, \hat{x}_2||h_2|) = Q(\frac{|h_2|\psi_2}{\varsigma_2}), \tag{18}$$

where $\psi_2 = \sqrt{p_1}|\Delta_2'|^2$ and $\varsigma_2 = \sqrt{2}\sigma_n|\Delta_2'|$.

Then, the exact expression of PEP of U_2 is

$$P(x_2, \hat{x}_2) = \frac{1}{2}(1 - \frac{\psi_2\delta_2}{\sqrt{\varsigma_2^2 + \psi_2^2\delta_2^2}}). \tag{19}$$

3.3. BER union bound

It is worth noting that the PEP depends on the transmitted and detected symbols of the users. Hence it should be averaged over all possible symbols of users. Given that x_l is transmitted symbol and $\hat{x}_l$ is the erroneously detected symbol, the BER union bound of the l-th user can be evaluated as

$$P_e \leq \tfrac{1}{b}\sum_{x_l}\Pr(x_l)\sum_{\substack{x_l \neq \hat{x}_l,\, l \neq p,\\ p \in \{1,2,\ldots,L\}}} q(x_l \rightarrow \hat{x}_l)\Pr(x_l \rightarrow \hat{x}_l) \tag{20}$$

where b is number of transmitted bits in x_l, $\Pr(x_l)$ is the probability of transmitting symbol x_l and $q(x_l \rightarrow \hat{x}_l)$ is the number of bit error when choosing $\hat{x}_l$ instead of x_l.

4. Asymptotic Analysis

In this section, we study the diversity gain based on the derived PEP. The diversity gain, d_l, is defined as the magnitude of the PEP slope for U_l when the signal-to-noise ratio (SNR) value reaches infinity.

4.1. Asymptotic Analysis for U_1

4.1.1. Ideal self-interference cancellation

Using the chernoff bound for the Q-function, under ideal self-interference cancellation, the conditional PEP expression in (7) can be bounded above by

$$P_i(x_1, \hat{x}_1|\Omega, |h_1|) \leq \exp(-\frac{\gamma_1 \psi_1^2}{4|\Delta_1|^2}). \tag{21}$$

where $\gamma_1 = \frac{|h_1|^2}{\sigma_n^2}$ is the instantaneous SNR, which is modeled as exponential random variable with PDF $f(\gamma_x) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma_x}{\bar{\gamma}}\right)$. At high SNR values, we have $\exp(-\frac{\gamma_1}{\bar{\gamma}}) \approx 1 - \frac{\gamma_1}{\bar{\gamma}}$. Therefore, from (21), we use binomial expansion and identity in [22, eq. 337.3.326.2.[10]] to obtain

$$P_i(x_1, \hat{x}_1|\Omega) \leq \frac{1}{\bar{\gamma}} \int_0^\infty \exp(\frac{-\gamma_1}{\bar{\gamma}}) \exp(\frac{-\gamma_1 \psi_1^2}{4|\Delta_1|^2}) \mathrm{d}\gamma = \sum_{k=0}^{1} (-1)^{1-k} \bar{\gamma}^{-2+k} \Gamma(2-k) (\frac{4|\Delta_1|^2}{\psi_1^2})^{2-k}. \tag{22}$$

When self-interference is completely eliminated, the probability that U_1 can successfully decode x_2 becomes

$$\Pr(\Omega) = \exp(-\frac{\gamma_2}{\bar{\gamma} p \beta}). \tag{23}$$

Therefore, under ideal conditions, the asymptotic unconditional PEP of U_1 can be evaluated as

$$P_i(x_1, \hat{x}_1) \leq \sum_{k=0}^{1} (-1)^{1-k} \bar{\gamma}^{-2+k} \Gamma(2-k) (\frac{4|\Delta_1|^2}{\psi_1^2})^{2-k} \exp(-\frac{\gamma_2}{\bar{\gamma} p \beta}) + \frac{1}{2}(1 - \exp(-\frac{\gamma_2}{\bar{\gamma} p \beta})). \tag{24}$$

4.1.2. Non-ideal self-interference cancellation

The conditional PEP in (7), under ideal self-interference cancellation, can be bounded above as

$$P_{ni}(x_1, \hat{x}_1||h_1|, \Omega) \leq \exp\left(-\frac{\psi_1^2 \gamma_1 + 2\sqrt{k} \psi_1 \psi_s \sqrt{\gamma_1} \sqrt{\gamma_s} + k \psi_s^2 \gamma_s}{4|\Delta_1|^2}\right), \tag{25}$$

where $\gamma_s = \frac{|h_s|^2}{\sigma_n^2}$.

By averaging (25) on h_1 and make variable substitute, $\gamma_1 = x^2$, $\gamma_s = y^2$, we can get (26) after some manipulations using [22, eq. 365.3.462.5.[11]], [22, eq. 346.3.381.9.*], and [22, eq.

337.3.326.2.[10]].

$$P_{ni}(x_1,\hat{x}_1) \leq \frac{16|\Delta_1|^4}{\left(\psi_1^2\bar{\gamma}+4|\Delta_1|^2\right)\left(k\psi_s^2\bar{\gamma}+4|\Delta_1|^2\right)} - \frac{32\sqrt{\pi}|\Delta_1|^4}{k\psi_1^2\psi_s^2\bar{\gamma}} \int_0^\infty x^2 \exp\left(-\xi_2^2 x^2\right)\left[1-\Phi\left(x\right)\right]dx \quad . \tag{26}$$

where

$$\xi_2 = \sqrt{\frac{(\psi_1^2\bar{\gamma}+4|\Delta_1|^2)(k\psi_s\bar{\gamma}+4|\Delta_1|^2)}{k\psi_1^2\psi_s\bar{\gamma}^2} - 1}. \tag{27}$$

By using the equation [22, eq. 647.6.292], inequality (26) can be further calculated as

$$P_{ni}(x_1,\hat{x}_1) \leq \frac{16|\Delta_1|^4}{(\psi_1^2\bar{\gamma}+4|\Delta_1|^2)(k\psi_s^2\bar{\gamma}+4|\Delta_1|^2)} - \frac{16|\Delta|^4}{k\psi_1^2\psi_s^2\bar{\gamma}^2}\left[\frac{\arctan(\xi_2)}{\xi_2^3} - \frac{1}{\xi_2^2(\xi_2^2-1)}\right]. \tag{28}$$

When self-interference is not completely eliminated, based on (9), the probability that U_1 successfully decodes x_2 can be expressed as

$$\Pr(\Omega) = \frac{p\beta}{\gamma_2 k p_1 + p\beta}\exp(-\frac{\gamma_2}{\bar{\gamma}p\beta}). \tag{29}$$

Therefore, under non-ideal conditions, the asymptotic unconditional PEP of U_1 can be evaluated as (30), which is at the top of the this page.

$$\begin{aligned} P_{ni}\left(x_1,\hat{x}_1\right) \leq & \left(\frac{16|\Delta_1|^4}{\left(\psi_1^2\bar{\gamma}+4|\Delta_1|^2\right)\left(k\psi_s^2\bar{\gamma}+4|\Delta_1|^2\right)} - \frac{16|\Delta_1|^4}{k\psi_1^2\psi_s^2\bar{\gamma}^2}\left(\frac{\arctan\xi_2}{\xi_2^3} - \frac{1}{\xi_2^2\left(\xi_2^2-1\right)}\right)\right)\frac{p\beta}{R_2kp_1+p\beta}\exp\left(-\frac{\gamma_2}{\bar{\gamma}p\beta}\right) \\ & + \frac{1}{2}\left(1 - \frac{p\beta}{R_2kp_1+p\beta}\exp\left(-\frac{\gamma_2}{\bar{\gamma}p\beta}\right)\right). \end{aligned} \tag{30}$$

4.2. Asymptotic Analysis for U_2

According to (22), the asymptotic unconditional PEP of U_2 can be evaluated as (31),

$$P\left(x_2,\hat{x}_2\right) \leq \sum_{k=0}^{1}\left(-1\right)^{1-k}\left(\bar{\gamma}\right)^{-2+k}\Gamma\left(2-k\right)\left(\frac{4\left|\Delta_2'\right|^2}{\psi_2^2}\right)^{2-k}. \tag{31}$$

Remark 1: From (24) and (31), we can easily obtain that the effective diversity orders for U_1 and U_2 under ideal case are both 1.

5. Numerical And Simulation Results

This section follows the aformentioned system model. We assume that the transmitted signals are randomly selected from the binary phase shift keying (BPSK) constellation. Unless otherwise specified, the simulation parameters are set as follows: power allocation coefficients $\alpha_1 = 0.3$, $\alpha_2 = 0.7$, and when $\alpha_1 \neq 0.3$, $\alpha_2 = 1 - \alpha_1$. Unless mentioned otherwise, the relay forwarding power $p_1 = p$, the threshold γ_2 is 1, the variance of the Rayleigh channel gain δ_i and δ_s is 0 dB, and the noise variance σ_n^2 is 0 dB. It is noted that x_i, $\hat{x}_i$ can be either $+1$ or -1 in BPSK modulation, resulting in $|\Delta_i| = 2$.

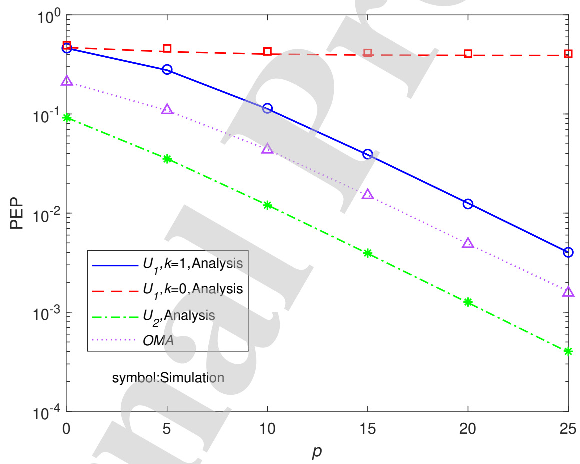

Figure 2: Analysis and simulated PEP for the 2 users under both ideal and non-ideal case with ipSIC.

Fig.2 shows the comparison of PEP of U_1 and U_2 in the FD cooperative relay downlink NOMA system, where $k = 0$ represents the ideal relay forwarding signal, $k = 1$ represents the influence of self-interference in the relay forwarding signal. It can be seen that the PEP of the two users decrease as the SNR increases. Also, the PEP for U_1 has nearly a stable value under the non-ideal case. We also show that the PEP of U_1 under ideal conditions is greatly lower than that the presence of self-interference, thereby indicating that self-interference has a great influence on the PEP of U_1, especially at high SNR slope.

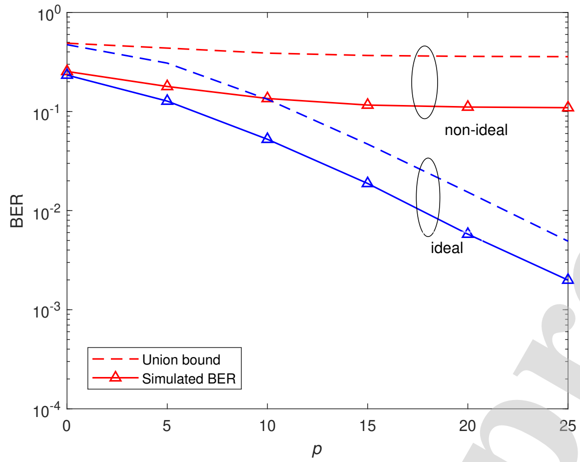

Figure 3: BER for U_1 under both ideal and non-ideal case with ipSIC.

Fig.3 shows the BER union bound of U_1 under the ideal case and the non-ideal case. It can be seen that the BER of U_1 decreases as the SNR increase, and the PEP is always the upper bound of the BER, it's because the PEP is averaged over all possible codewords of all users. Moreover, it can be noticed from the figure that the PEP can provide an indication about the diversity order, where the slope of the union bound matches the slope of the BER at high SNR.

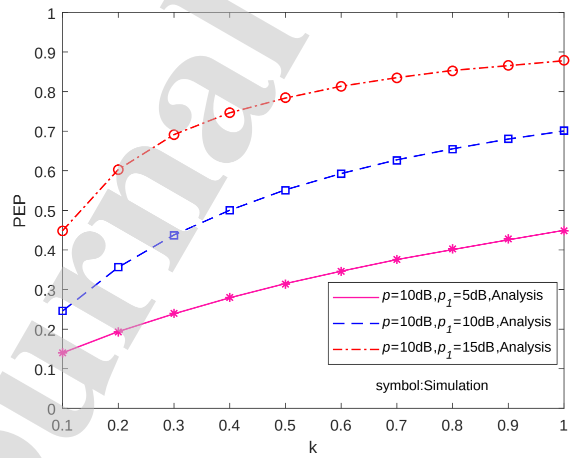

Figure 4: PEP for U_1 over different self-interference coefficient.

Fig.4 plots the PEP for U_1 versus self-interference coefficient k. It can be seen that the PEP of U_1 increases as k increases, which further verifies the observation that the PEP of U_1 is greatly influenced by self-interference. The PEP of relay user U_1 increases with the increase of p_1, thereby

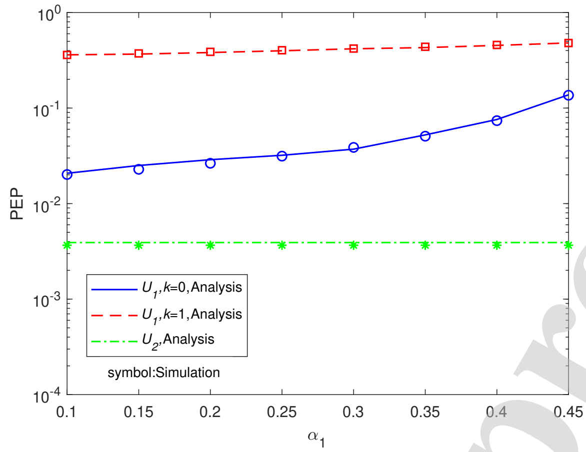

Figure 5: PEP for 2 users over different power allocations.

showing that PEP performance of U_1 degrade as the transmit power p_1 increases due to self-interference.

Fig.5 plots the PEP for both U_1 and U_2 versus power allocation parameter α_1. It can be seen that the PEP of U_1 increase with the increase of α_1, which is due to the fact that the successful decoding probability to detect U_2 at U_1 will decrease with the increase of α_1. Moreover, the performance gap between ideal case and non-ideal case for U_1 is nearly the same when $\alpha_1 < 0.4$. However, the power allocation parameter has negligible impact on the PEP of U_2.

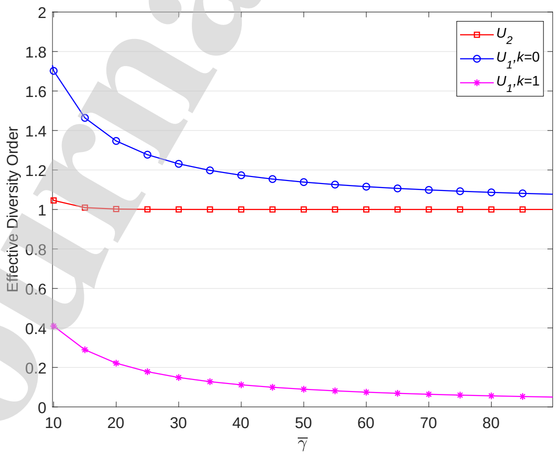

Figure 6: Effective diversity order for the 2 users under both ideal and non-ideal case.

Fig.6 shows the effective diversity order of the considered system. At high SNR, the diversity order for both U_1 under ideal conditions and U_2 converges to 1, while the diversity order of U_1 under non-ideal conditions converges to zero, which illustrates that the SI induced by imperfect FD has great impact on the effective diversity order of the relay node (i.e. U_1).

6. Conclusion

In this paper, the BER performance of downlink full-duplex cooperative relay NOMA system in terms of PEP is studied by taking into account the impact of SI under the assumption of ipSIC. Considering the two-user NOMA scheme, The closed-form and asymptotic expressions of PEP are derived for both ideal and non ideal scenarios. Numerical and analytical results show that SI has different effects on the near user and far-away user's PEP performance, which is different from the reported results on the outage performance. In addition, the diversity order is sharply decreased in non-ideal case compared with ideal case for the far-away user.

Acknowledgment

This work was supported in part by the National Natural Science Foundation of China (No. 62001320), the Key Research and Development Program of Shanxi (No. 201903D121117), Research Project Supported by Shanxi Scholarship Council of China (No. 2021-133 and 2020-126), Graduate Education Innovation Project of Shanxi (No. 2019SY250 and 2020SY419).

Declaration of competing interest

The authors declared that they had no conflicts of interest with respect to their authorship or the publication of this article.

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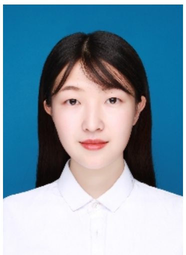

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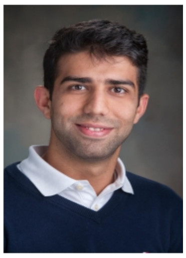

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Meiling Li and Sami Muhaidat: Conceptualization, Methodology. **Meiling Li, Shuaibo Huang and Lili Tian**: Investigation, Software, Data curation and Writing Original draft preparation; **Meiling Li and Shuaibo Huang:** Supervision; **Omar Alhussein:** Visualization. **Meiling Li, Shuaibo Huang, Lili Tian, Omar Alhussein and Sami Muhaidat:** Writing-Reviewing and Editing; **Meiling Li, Shuaibo Huang:** Address comments.

Declaration of interests

☒ The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

☐The authors declare the following financial interests/personal relationships which may be considered as potential competing interests: