Performance Analysis of Energy Detection over Mixture Gamma based Fading Channels with Diversity Reception

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Introductory Background | Fading Channels

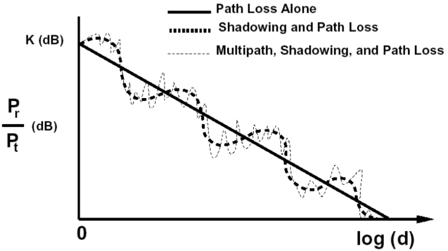
This paper is concerned with providing a unified approach to analyzing the performance of cognitive radio networks.

- Large-scale (Shadowing) Effect
- Lognormal, Inverse-Gaussian, Gamma.

Introductory Background | Fading Channels

This paper is concerned with providing a unified approach to analyzing the performance of cognitive radio networks.

- Large-scale (Shadowing) Effect
- Lognormal, Inverse-Gaussian, Gamma.
- Small-scale (Multipath) Effect
- **Conventional**: Rayleigh, Nakagami-m, Weibull-m, Rician.
- **Generalized**: $\alpha - \mu$, $\kappa - \mu$, $\eta - \mu$.



Introductory Background | Composite Fading Channels (1)

Performance analysis over multipath or shadowing scenarios alone is relatively tractable. However, extending the analysis to composite fading channels is rather cumbersome and intractable.

- Examples:
  - Nakagami-m/Lognormal (NL) model (no closed-form)

$$f_{\gamma}(x) = \frac{2\lambda m^m}{\Gamma(m)\sqrt{2\pi}\zeta} \int_0^{\infty} \frac{x^{m-1}}{\overline{\gamma}^m \sigma^{m+1}} e^{-\frac{mx}{\overline{\gamma}\sigma}} e^{\frac{-(10\log\sigma)^2}{2\zeta^2}} \mathrm{d}\sigma. \qquad (1)$$

  - $\kappa - \mu$ Shadowed model [1] (complicated)

$$f_{\gamma}(\gamma) = \frac{\mu^{\mu} m^m (1+\kappa)^{\mu}}{\Gamma(\mu)\overline{\gamma}(\mu\kappa+m)^m} (\frac{\gamma}{\overline{\gamma}})^{\mu-1} \qquad (2)$$
$$\times \exp(-\frac{\mu(1+\kappa)\gamma}{\overline{\gamma}})\, {}_1\mathcal{F}_1(m,\mu;\frac{\mu^2\kappa(1+\kappa)}{\mu\kappa+m}\frac{\gamma}{\overline{\gamma}}),$$

[1] J. F. Paris, "Statistical Characterization of κ-μ Shadowed Fading," IEEE Trans. Veh. Technol., vol. 63, no. 2, pp. 518–526, Feb. 2014.

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  - The K- and K_G- distributions:
    - Replaced the Lognormal with Gamma and integrate.
  - The RIGD and G-distribution:
    - Replaced the Lognormal with Inverse-Gaussian (IG) and integrate.
- Still complex and not a general solution.

Introductory Background | Composite Fading Channels (3)

- Another alternative proposed by Atapattu *et al.* [2]
  - Several models were approximated using the mixture Gamma (MG) distribution via gauss-quadrature approximations.
  - Approximates: NL, K, K_G, $\eta - \mu$, $\kappa - \mu$, Hoyt, and Nakagami-m.
- The pdf is given by

$$f_\gamma(\gamma) = \sum_{i=1}^{K} \frac{\alpha_i}{\overline{\gamma}} \frac{x}{\overline{\gamma}}^{\beta_i - 1} \exp(\frac{\zeta_i x}{\overline{\gamma}})$$

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  - Pros: Very tractable, arbitrarily accurate.
  - Cons: Still not generalizable to all fading models.

[2] S. Atapattu, C. Tellambura, and H. Jiang, "A Mixture Gamma Distribution to Model the SNR of Wireless Channels," IEEE Trans. Wireless Commun., vol. 10, no. 12, pp. 4193–4203, Dec. 2011.

Introductory Background | Energy Detection Performance

- Cognitive radios came as a viable solution to mitigate the spectrum scarcity.
- Spectrum sensing techniques:
  - Energy Detector (ED).
  - Others: Matched Filter, cyclostationarity or feature detection .
- Here we are concerned with the performance of the ED in generalized and composite fading channels.

Introductory Background | Energy Detection Performance

- Approaches in literature:
  - A semi-analytic solution for the NL was proposed.
  - The K- and K_G- distribution is widely utilized to study the ED performance over RL channels; yet analytically cumbersome.
  - The Mixture of Gaussian (MoG) distribution [3] was utilized to study the ED performance.
- Likewise, here we utilize the mixture distributions approach, i.e.
  - We utilize the MG distribution to study the ED performance.

[3] O. Alhussein, O. Selim, T. Assaf, S. Muhaidat, J. Liang, and G. K. Karagiannidis, "A Generalized Mixture of Gaussians Model for Fading Channels," in IEEE Conf. Vehicul. Tech., Glasgow, 2015.

Energy Detection Performance

- Goal is to study the detection and false-alarm probabilities in MG-based generalized and composite fading channels.
- We consider square-law combining (SLC) and square-law selection (SLS) diversity schemes.
- Typical configuration:

$$\begin{aligned} \mathcal{H}_0 &: y_j(t) = h_j(t) \\ \mathcal{H}_1 &: y_j(t) = h_j(t) + v_j(t)\,, \end{aligned} \tag{3}$$

- This conditional detection and false-alarm probabilities:

$$P_d = Q_u(\sqrt{2\gamma_j}, \sqrt{\lambda_n}), \tag{4}$$

$$P_f = \frac{\Gamma(u, \frac{\lambda_n}{2})}{\Gamma(u)}, \tag{5}$$

where u is the time-bandwidth product, λ_n is the ED threshold.

Single-antenna Scenario (1)

- Our generelized fading model is written as

$$f_\gamma(x) = \sum_{k=1}^{K} \frac{\alpha_k}{\gamma_0} \left(\frac{x}{\gamma_0}\right)^{\beta_k - 1} \exp\left(-\frac{\zeta_k x}{\gamma_0}\right) . \qquad (6)$$

- Consequently, the detection probability is computed as

$$\overline{P}_{d,MG} = \sum_{k=1}^{K} \frac{\alpha_k}{\gamma_0} \int_0^\infty Q_u(\sqrt{2x}, \sqrt{\lambda_n}).(\frac{x}{\gamma_0})^{\beta_k - 1} e^{-\frac{\zeta_k x}{\gamma_0}} dx . \qquad (7)$$

Single-antenna Scenario (2)

- Using Theorem 1 in [4], we obtain

$$\overline{P}_{d,MG} = \sum_{k=1}^{K} \frac{\alpha_k \Gamma(\beta_k)\Gamma(u, \frac{\lambda_n}{2})}{\Gamma(u)\zeta_k^{\beta_k}} + \sum_{k=1}^{K}\sum_{l=0}^{\beta_k-1} \frac{\alpha_k \Gamma(\beta_k)}{\gamma_0^{\beta_k}} \times \frac{\left(\frac{\lambda_n}{2}\right)^u {}_1\mathcal{F}_1(l+1, u+1, \frac{\lambda_n/2}{1+1\frac{\zeta_k}{\gamma_0}})}{u!(\frac{\zeta_k}{\gamma_0})^{\beta_k-l}(1+\frac{\zeta_k}{\gamma_0})^{l+1}\exp(\frac{\lambda_n}{2})}, \quad (8)$$

[4] P. Sofotasios, M. Valkama, T. Tsiftsis, Y. Brychkov, S. Freear, and G. Karagiannidis, "Analytic solutions to a Marcum Q-function-based integral and application in energy detection of unknown signals over multipath fading channels," in IEEE Cogn. Radio Oriented Wireless Netw. and Commun. (CROWNCOM), 2014, pp. 260–265.

Square-Law Combining (1)

- Under SLC, the received signals from each branch are integrated, squared, and then summed up.

$$\gamma_{\Sigma} = \sum_{l=1}^{L} \gamma_l \, . \tag{9}$$

- We need to obtain pdf of γ_{Σ} in order to derive $P_{d,\Sigma}$.

$$\begin{aligned} f_{\gamma_{\Sigma}}^{(2)} &= \int_0^{\gamma} f_{\gamma_1}(x) f(\gamma - x) dx = \sum_{i=1}^{K} \sum_{j=1}^{K} \frac{\alpha_i \alpha_j}{{\gamma_0}^{\beta_i + \beta j}} \\ &\times \int_0^{\gamma} x^{\beta_i - 1} e^{-\frac{\zeta_i}{\gamma_0} x} (\gamma - x)^{\beta_j - 1} e^{-\frac{\zeta_j}{\gamma_0}(\gamma - x)} dx. \end{aligned} \tag{10}$$

Square-Law Combining (2)

- Under SLC, the received signals from each branch are integrated, squared, and then summed up.

$$\gamma_\Sigma = \sum_{l=1}^{L} \gamma_l \, . \tag{11}$$

- The pdf of γ_Σ was derived

$$f_{\gamma_\Sigma}^{(2)}|_{(\zeta_i=\zeta_j)} = \sum_{i=1}^{K}\sum_{j=1}^{K} \frac{\alpha_i\alpha_j}{{\gamma_0}^{\beta_i+\beta_j}} \frac{\Gamma(\beta_i)\Gamma(\beta_j)}{\Gamma(\beta_i+\beta_j)} e^{-\frac{\zeta_j}{\gamma_0}\gamma} \gamma^{\beta_i+\beta_j-1}. \tag{12}$$

Square-Law Combining (3)

$$f_{\gamma_\Sigma}^{(2)}|_{(\zeta_i \neq \zeta_j)} = \sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{l=0}^{\beta_j-1}\binom{\beta_j-1}{l}\frac{(-1)^l\alpha_i\alpha_j}{{\gamma_0}^{\beta_j-l}(\zeta_i-\zeta_j)^{\beta_i+l}} \times \gamma^{\beta_j-l-1}e^{-\frac{\zeta_j}{\gamma_0}\gamma}\gamma\left(\beta_i+l,\frac{\gamma(\zeta_i-\zeta_j)}{\gamma_0}\right), \quad (13)$$

while, $f_{\gamma_\Sigma}^{(2)} = f_{\gamma_\Sigma}^{(2)}|_{(\zeta_i=\zeta_j)} + f_{\gamma_\Sigma}^{(2)}|_{(\zeta_i\neq\zeta_j)}$.

- Note: This method works for higher diversity orders.

Square-Law Combining (4)

- Thus $\overline{P}_{d,\Sigma_2}$ was derived as

$$\overline{P}^{(2)}_{d,\Sigma}|_{(\zeta_i=\zeta_j)} = \sum_{i=1}^{K}\sum_{j=1}^{K}\alpha_i\alpha_j\Gamma(\beta_i)\Gamma(\beta_j)\left[\frac{\Gamma(u,\frac{\lambda}{2})}{\zeta_j^{\beta_i+\beta_j}\Gamma(u)} + \sum_{n=0}^{\beta_i+\beta_j-1}\frac{\gamma_0{}^{-n}(\frac{\lambda}{2})^u\,{}_1\mathcal{F}_1(n+1,u+1,\frac{\frac{\lambda}{2}}{1+\frac{\zeta_j}{\gamma_0}})}{u!(\zeta_j)^{\beta_i+\beta_j-n}(1+\frac{\zeta_j}{\gamma_0})^{n+1}\exp(\frac{\lambda}{2})}\right] \quad (14)$$

Square-Law Combining (5)

$$\overline{P}_{d,\Sigma}^{(2)}|_{(\zeta_i \neq \zeta_j)} = \sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{l=0}^{\beta_j-1}\binom{\beta_j-1}{l}\frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)}{{\gamma_0}^{\beta_j-l}(\zeta_i-\zeta_j)^{\beta_i+l}} \tag{15}$$

$$\times\left[\frac{\Gamma(\beta_j-l)\Gamma(u,\frac{\lambda}{2})}{(\frac{\zeta_j}{\gamma_0})^{\beta_j-l}\Gamma(u)}+\sum_{n=0}^{\beta_j-l-1}\frac{(\frac{\lambda}{2})^u\Gamma(\beta_j-l)}{u!\,(\frac{\zeta_j}{\gamma_0})^{\beta_j-l-n}}\frac{{}_1\mathcal{F}_1(n+1,u+1,\frac{\frac{\lambda}{2}}{1+\frac{\zeta_j}{\gamma_0}})}{(1+\frac{\zeta_j}{\gamma_0})^{n+1}\exp(\frac{\lambda}{2})}\right]$$

$$-\sum_{i=1}^{K}\sum_{j=1}^{K}\sum_{l=0}^{\beta_j-1}\sum_{t=0}^{\beta_i+l-1}\binom{\beta_j-1}{l}\frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)}{t!{\gamma_0}^{\beta_j-l+t}(\zeta_i-\zeta_j)^{\beta_i+l-t}}$$

$$\times\left[\frac{\Gamma(\beta_j+t-l)\Gamma(u,\frac{\lambda}{2})}{(\frac{\zeta_i}{\gamma_0})^{\beta_j+t-l}\Gamma(u)}+\sum_{n=0}^{\beta_j+t-l-1}\frac{(\frac{\lambda}{2})^u\Gamma(\beta_j+t-l)\,{}_1\mathcal{F}_1(n+1,u+1,\frac{\frac{\lambda}{2}}{1+\frac{\zeta_i}{\gamma_0}})}{u!\,(\frac{\zeta_i}{\gamma_0})^{\beta_j+t-l-n}(1+\frac{\zeta_i}{\gamma_0})^{n+1}\exp(\frac{\lambda}{2})}\right].$$

Square-Law Selection (1)

- Under SLS, the branch with the maximum γ_j is selected as follows

$$\gamma_{\mathrm{SLS}} = \max_{j=1,..,L} (\gamma_j) \,. \tag{16}$$

- Under $\mathcal{H}_0$, the false-alarm probability for the SLS scheme can be expressed as

$$P_{f,\mathrm{SLS}} = 1 - \Pr(\gamma_{\mathrm{SLS}} < \lambda_n | \mathcal{H}_0) \,. \tag{17}$$

- Substituting (16) in (17), we obtain

$$P_{f,\mathrm{SLS}} = 1 - \Pr(\max(\gamma_1, \gamma_2, .., \gamma_L) < \lambda_n | \mathcal{H}_0) \,. \tag{18}$$

Square-Law Selection (2)

- Accordingly, this translates to

$$P_{f,\mathrm{SLS}} = 1 - \left[1 - P_f\right]^L . \tag{19}$$

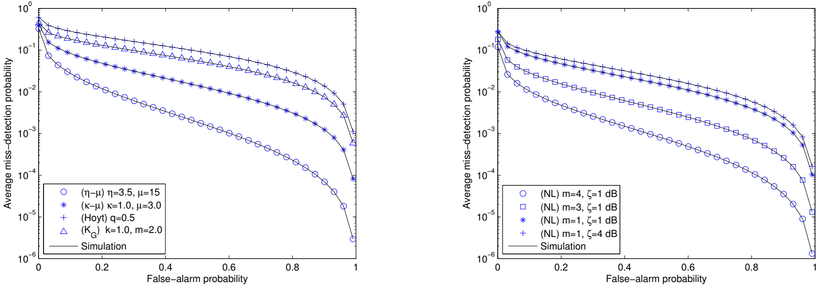
- Similarly, the unconditional probability of detection over the AWGN channel is obtained by

$$P_{d,\mathrm{SLS}} = 1 - \prod_{j=1}^{L} \left[1 - Q_u(\sqrt{2\gamma_j}, \sqrt{\lambda_n})\right] . \tag{20}$$

- Hence, averaging (20) over (7) yields the unconditional probability of detection under the SLS scheme, $\bar{P}_{d,\mathrm{SLS}}$, which is given by

$$\bar{P}_{d,\mathrm{SLS}} = 1 - \prod_{j=1}^{L} \left[1 - P_{d,MG}\right] . \tag{21}$$

Simulation Results



(Left) depicts $1 - P_d$ versus P_{fa} (ROC) curves for different fading conditions with no diversity. (Right) depicts the ROC curves over several scenarios of the composite NL fading channel with SLC diversity scheme with $L = 2$.

Conclusions

- We proposed a unified framework for the performance analysis of an energy detector in generalized and composite MG-based fading channels.
- Novel analytical expressions for the average detection probability have been derived for both the single-antenna case and the multiple-antenna case with SLC and SLS schemes.
  - Derived expressions are **generalized** in terms of fading characterization and **algebraically versatile**.
- Notes:
  - This paper can be found in the ArXiv.orgrepository (arXiv:1510.05594).
  - An extended journal edition of the paper is found as an *early-access* IEEE TVT transaction.