Unified Analysis of Diversity Reception in the Presence of Impulsive Noise

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***Abstract*—We consider a single-input-multiple-output (SIMO) system over generalized and composite fading channels using the mixture Gamma (MG) distribution in the presence of impulsive noise, which is modeled by Middleton's Class-A (MCA) and ϵ-mixture noise models. First, we develop a simple and effective information theoretic approach to determine the optimal number of components for the MG distribution based on the Bayesian information criterion. We then derive novel pairwise error probability (PEP) expressions for the considered system with maximal-ratio combining and selection combining at the receiver. The derived PEP expressions involve finite single-fold integrals, which are further simplified to rather more tractable expressions applicable for the special case of integer values of the scale parameter β_k. Furthermore, we provide analytical tractable expressions for the average channel capacity under the impulsive noise assumption for the considered system. Analytical and Monte Carlo simulation analysis are presented to validate the analytical results.**

***Index Terms*—Generalized fading channels, impulsive noise, pairwise error probability, SIMO**

I. Introduction

In a typical mobile radio environment, the received signal undergoes small-scale power fluctuations (microscopic fading) superimposed on large-scale signal power fluctuations (shadowing or macroscopic fading). When both macroscopic and microscopic fading are present, they can be modeled by composite fading distributions. A popular example of such composite fading channels is the Nakagami/Lognormal (NL), where the density function is obtained by averaging the instantaneous Nakagami-m fading average power over the conditional probability density function (PDF) of the Lognormal shadowing. However, this results in a complicated expression for the PDF that has no closed form [1].

Several generalized distributions are proposed to model or approximate composite fading channels, such as the K-distribution [2], K_G-distribution [3], Rayleigh/Inverse-Gaussian (RIGD) [4], G-distribution [5], Weibull/Gamma [6], $\kappa-\mu$/Gamma [7], $\alpha-\kappa-\mu$ extreme distribution [8], mixture of Gaussian (MoG) [9], et al. The mixture Gamma (MG) distribution, proposed in [10] by Attapattu et al., accurately approximates several generalized and composite fading models via Gaussian quadrature approximations. Performance metrics based on the MG distribution are often analytically tractable, offering closed-form solutions to support analysis. Moreover, high approximation accuracy can be achieved by increasing the number of summed mixture components.

Diversity techniques, which exploit multiple copies of the transmitted signal, have been widely investigated to overcome detrimental effects of wireless channels. Most of the reported results in the literature assume additive white Gaussian noise (AWGN) in each diversity branch. Although this assumption incorporates the effect of background thermal noise, it ignores the impact of the impulsive noise caused by atmospheric, man-made partial discharge, switching effect, and electromagnetic interference, et cetera, [11], [12]. The Middleton's Class-A (MCA) model [13] is one of the most accurate statistical-physical models for narrowband impulsive noise. There are many works in the literature that employs the MCA model to characterize impulsive noise in wireless communication systems, c.f., [11], [12], [14]–[19]. Expressions for the bit error probability (BEP) in selection combining (SC) and maximal ratio combining (MRC) receivers over Rayleigh fading channel with MCA noise were derived in [20]. In [11], the same channel model was considered to derive the BEP for different combining schemes, including SC and MRC. The BEP over Rician channel in the presence of MCA noise was derived for both SC and equal gain combining (EGC) in [15]. Moreover, since the MCA model contains an infinite number of noise states, a relatively tractable model, widely known as ϵ-mixture noise model, with two terms and two tunable parameters is considered in [14], [21]. Although there have been considerable research efforts on diversity analysis over conventional multipath fading channels, such as Rayleigh and Nakagami-m, to the best of our knowledge, there exist no reported results that incorporates generalized or composite fading channels along with MCA or ϵ-mixture noise.

In this paper, we aim to fill this research gap and investigate the performance of single-input-multiple-output (SIMO) system over generalized and composite fading channels with MCA and ϵ-mixture noise. Following [10] and [22], we

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This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2016.2569064, IEEE Transactions on Vehicular Technology

express numerous generalized and composite fading channels, such as the NL, K_G, $\eta-\mu$, $\kappa-\mu$, Lognormal, and Weibull, by the MG distribution. The contributions in this paper are multifold, and can be summarized as follows:

- We propose a simple and effective information theoretic approach to determine the optimal number of components for the MG distribution based on the Bayesian information criterion (BIC).
- We derive analytical pairwise error probability (PEP) expressions that involve finite single-fold integrals assuming MCA and ϵ-mixture noise with MRC and SC. Furthermore, more tractable PEP expressions are derived for the case of integer values of the involved parameter β_k.
- We derive closed-form expressions for the average channel capacity assuming MCA and ϵ-mixture noise with MRC and SC.

The remainder of the paper is organized as follows: Section II provides a brief introduction to the two impulsive noise models. In Section III, we introduce the MG distribution. Then, the BIC is proposed as an approach to determine the optimal number of components for numerous MG-based generalized and composite fading models. The SIMO system model with MCA and ϵ-mixture noise, followed by the derivation PEP and average channel capacity expressions, are introduced in Section V. Analytical and Monte Carlo simulations are presented in Section VI, followed by some concluding remarks in Section VII.

II. Middleton's Class-A and ϵ-Mixture Impulsive Noise Models

Following [11], we assume that the noise at the receiver is modeled as $n = n_g + n_i$, where n_g, n_i are the background Gaussian noise with variance σ_g^2, and the impulsive noise with variance σ_i^2, respectively. When the number of independent active sources, namely K, is large enough, the occurrences of interference would follow a Poisson distribution, i.e.

$$\Pr(K=k) = \frac{e^{-A}A^k}{k!}, \tag{1}$$

where A is the impulsive index that describes the average number of impulses during some interference time [11], and it is typically in the range of 10^{-4} to 0.5. As such, the PDF of the MCA noise can be expressed as [21]

$$\begin{aligned} f_n(x) &= \sum_{k=0}^{\infty} \Pr(K=k) f_{n|K=k}(x|k) \\ &= \sum_{k=0}^{\infty} \frac{e^{-A}A^k}{k!\sqrt{2\pi\sigma_k^2}} \exp(-\frac{x^2}{2\sigma_k^2}), \end{aligned} \tag{2}$$

In (2), $\sigma_k^2 = \frac{kA^{-1}+\Xi}{1+\Xi}\sigma^2$, where $\sigma^2 = \frac{N_0}{2}$ is the power of n, $\Xi = \frac{\sigma_g^2}{\sigma_i^2}$ is the Gaussian Factor, which resembles the ratio of the variances of the background Gaussian component the impulsive component, and it is typically in the range of 10^{-5} to 1. Although this distribution includes an infinite summation, it is completely described by three parameters, σ^2, Ξ, and A. As A and Ξ tends to zero, the noise becomes more impulsive. In the analysis hereafter, we truncate the MCA model to $C+1$ components. In [13], Zabin and Poor proposed an expectation-maximization (EM) based method to estimate Ξ and A. The method relies on the iterative maximization of the log-likelihood function of the envelope of the MCA noise, where the envelope is expressed as an infinite summation of weighted Rayleigh distributions.

Another popular impulsive noise model is the ϵ-mixture [21], which resembles a Gaussian-Gaussian mixture model, where the the PDF is expressed as

$$f_n(x) = \frac{1-\epsilon}{\sqrt{2\pi\sigma_g^2}} \exp(-\frac{x^2}{2\sigma_g^2}) + \frac{\epsilon}{\sqrt{2\pi\sigma_i^2}} \exp(-\frac{x^2}{2\sigma_i^2}), \tag{3}$$

where ϵ denotes the fraction of time for which the impulsive noise occurs, with $0 < \epsilon < 1$. The ratio of the variances of the impulsive component to the Gaussian component is given by $\xi = \frac{\sigma_i^2}{\sigma_g^2}$. Here, the power of n is given by

$$\sigma^2 = \frac{N_0}{2} = (1-\epsilon)\sigma_g^2 + \epsilon\sigma_i^2. \tag{4}$$

In this context, given a reference noise process, Gaussian mixture based EM estimation technique can also be utilized for the estimation of ϵ, σ_g^2, and σ_i^2 [9]. It is worth noting that although a truncated two-term MCA model has been shown to accurately represent the infinite MCA noise model when A and Ξ are small enough, the truncated two-term MCA model remains as a subset of the ϵ-mixture noise model, as shown in [21].

III. Approximating Fading Models with the MG Distribution

The MG distribution was shown to accurately approximate numerous generalized and composite fading channels, such as the Nakagami/Lognormal (NL) [10, eq. (5)], K_G [10, eq. (8)], $\eta-\mu$ [10, eq. (10)], and $\kappa-\mu$ [10, eq. (17)]. The PDF of the MG distribution consists of a convex linear combination of Gamma distributions as

$$f_\gamma(x) = \sum_{j=1}^{G} \frac{\alpha_j}{\overline{\gamma}} (\frac{x}{\overline{\gamma}})^{\beta_j-1} \exp(-\zeta_j \frac{x}{\overline{\gamma}}), \tag{5}$$

where G denotes the number of mixture components, α_j, $j = 0,..,G$, is the mixing coefficient of the jth component having the constraints $0 \leq \frac{\alpha_j\Gamma(\beta_j)}{\zeta_j^{\beta_j}} \leq 1$ and $\sum_{j=1}^{G} \frac{\alpha_j\Gamma(\beta_j)}{\zeta_j^{\beta_j}} = 1$, where $\Gamma(.)$ is the gamma function [23, eq. (8.310.1)]. The scale and shape parameters of the jth component are β_j and ζ_j, respectively, while $\overline{\gamma} = \frac{E_b}{N_0} = \mathbb{E}[\gamma]$ is the average SNR per bit, where $\mathbb{E}[.]$ denotes expectation.

In this paper, we determine the appropriate number of components for the MG distribution. In [10], the number of components is selected manually such that the mean-square error (MSE) or Kullback-Leibler (KL) divergence between the actual distribution and the MG is below a predefined threshold. This method requires determination of a suitable target threshold empirically or by means of Monte Carlo simulations which may be a tedious task to accomplish. Choosing a small number of components yield an inaccurate representation, whereas a

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large number would unnecessarily increase the complexity of the distribution and may cause overfitting. Instead, we propose the use of a simple yet effective unsupervised criterion, named the Bayesian information criterion (BIC), which was introduced by Gideon Schwarz in [24].

Let $\boldsymbol{x} = \{x_1, ..., x_z, ..x_N\}$ correspond to N independent and identically distributed (*i.i.d.)* samples, drawn from any of the actual aforementioned SNR fading models, then the approximation methods used in [10] and [22] rely on the maximization of the likelihood function, $\Pr(\boldsymbol{x}|\hat{\theta}, G)$, where $\hat{\theta} = (\hat{\alpha}_1, \hat{\alpha}_2, ..., \hat{\alpha}_G, \hat{\beta}_1, \hat{\beta}_2, ..., \hat{\beta}_G, \hat{\zeta}_1, \hat{\zeta}_2, ..., \hat{\zeta}_G)$ are the estimated parameters and G is the corresponding number of components. The BIC is an asymptotic approximation to the transformation of the Bayesian *a posteriori* probability, $\Pr(\hat{\theta}|\boldsymbol{x}, G)$. As such, in a large-sample setting, the number of components determined by the BIC is optimal from the perspective of the Bayesian posterior probability. In addition, the BIC does not rely on the specification of prior distributions of $\hat{\theta}$, but only on the log-likelihood function, which can be readily achieved. For a candidate model with complexity labelled by G components, the log-likelihood function is expressed as

$$\mathbb{L}_{(MG)}(\hat{\theta}|\boldsymbol{x}, G) = \sum_{z=1}^{N} \ln\left[\sum_{i=1}^{G} \hat{\alpha}_i x_z^{\hat{\beta}_i - 1} \exp(-\hat{\zeta}_i x_z)\right]. \tag{6}$$

The corresponding BIC estimate can be computed as

$$BIC_G = -2\,\mathbb{L}_{(MG)}(\hat{\theta}|\boldsymbol{x}, G) + G\,\ln N. \tag{7}$$

It can be seen that the BIC penalizes the model complexity by adding the regularization coefficient, $G\,\ln N$. Here we select the candidate model satisfying the minimum BIC estimate or equivalently the asymptotically maximum Bayesian posterior probability as

$$G_{opt} = \arg\min_{G \in \mathbb{N}} BIC_G. \tag{8}$$

Fig. 1 depicts the BIC versus the number of components of various generalized and composite MG-based fading channels. The corresponding optimal number of components G_{opt}, indicated in the legend, will be adopted in the simulations and numerical results hereafter, and it will be denoted by G.

Another possible criterion is the Akaike information criterion (AIC) [25], which can be computed as

$$AIC_G = -2\mathbb{L}_{(MG)}(\hat{\theta}|\boldsymbol{x}, G) + 2G. \tag{9}$$

The AIC serves as an asymptotic unbiased estimator of the KL divergence between the actual distribution and the MG distribution. We point out that the BIC tends to be more penurious as compared to the AIC, i.e. the BIC favors model candidates with smaller G, since $G\,\ln N > 2K$ for $N > e^2$.

Here it merits a mention that the BIC approach does not add a considerable computational overhead to the adopted parameter estimation techniques, namely using Gauss-Hermite and Gauss-Laguerre methods [10]. Classically, the computation required for the weights and abscissas of the Gauss-Hermite and Gauss-Lageurre polynomials require $\mathcal{O}(n^2)$ operations [26], where $\mathcal{O}(.)$ is the big Oh notation, resembling an upper bound on the runtime of the algorithm [27]. In more recent

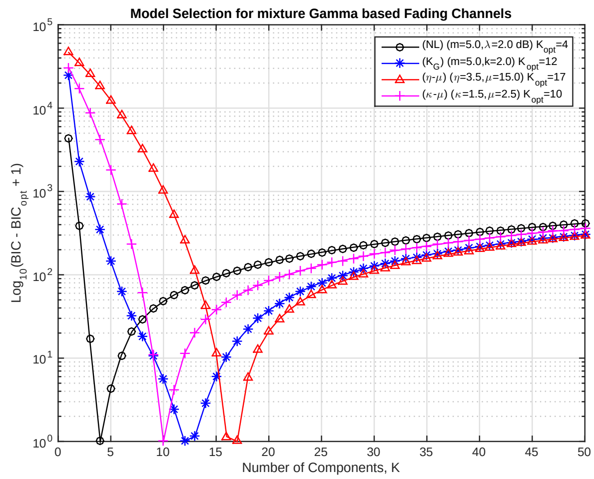

Figure 1. Normalized BIC versus number of components for various generalized and composite MG-based fading channels.

works, the computational complexity for the weights and abscissas of the respective polynomials is reduced to $\mathcal{O}(n)$ operations [28], [29]. Furthermore, the computation of the BIC measure for each G requires a only one vectorized computation as governed by (7).

IV. Performance Analysis

A. System Model

Consider a point-to-point communication scenario with a multi-link channel having L *i.i.d.* slowly varying and flat fading channels h_l, $l = 1, .., L$. The received signal copy via lth branch is given by

$$r_l = h_l\, s + n_l, \tag{10}$$

where $s \in \mathcal{S}$ is the transmitted symbol belongs to the constellation $\mathcal{S}$, with an average signal energy $E_b = \mathbb{E}[|s|^2] = 1$, n_l is the noise impaired in lth receiver branch, following the MCA noise model, i.e., $n_l \sim \sum_{k=0}^{\infty} \frac{e^{-A} A^k}{k!} \mathcal{CN}(0, \sigma_k^2)$, with $\sigma_k^2 = \frac{kA^{-1} + \Xi}{1+\Xi} \sigma^2$, where $\sigma^2 = N_0$ is the total noise variance[1]. Here the power gain, $\gamma_l = \overline{\gamma} h_l^2$, of the lth channel is modeled by MG distribution as discussed in Section III, where $\overline{\gamma} = \frac{E_b}{N_0}$. In the following subsections, we derive expressions for the PEP and average channel capacity for both MRC and SC schemes.

B. Pairwise Error Probability

Although the L_2-norm detection scheme is suboptimal for the case of impulsive noise, it is considered in our analysis due to its versatile application and low computational complexity. For a single branch, the L_2-norm detector selects the most likely symbol using

$$\hat{s} = \min_{\tilde{s} \in \mathcal{S}} ||r_l - h_l\, \tilde{s}||^2, \tag{11}$$

[1]Although, not explicitly derived, extending the herein analysis to ϵ-mixture noise is straightforward, where $n_l \sim (1-\epsilon)\,\mathcal{CN}(0, \sigma_1^2) + \epsilon\,\mathcal{CN}(0, \sigma_2^2)$ as in Section II, with $\sigma^2 = N_0 = (1-\epsilon)\sigma_g^2 + \epsilon\sigma_i^2$.

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where $\tilde{s} \in \mathcal{S}$ denotes a candidate symbol in the constellation $\mathcal{S}$.

1) Pairwise Error Probability for MRC: It follows that we can express the conditional PEP between any two symbols $\hat{s}$ and $s \in \mathcal{S}$ for the lth single branch as

$$P_{PEP}^{(l)}(d|\gamma_l) = \sum_{k=0}^{C} \frac{e^{-A}A^k}{k!} \Pr(\frac{||r-h\hat{s}||^2}{\sigma_k^2} < \frac{||r-hs||^2}{\sigma_k^2}), \tag{12}$$

where $C+1$ is the number of components in the MCA noise model, d denotes the Euclidean distance between s and $\hat{s}$. From (12), the conditional PEP of the lth branch is thus written as

$$P_{PEP}^{(l)}(d|\gamma_l) = \sum_{k=0}^{C} \frac{e^{-A}A^k}{k!\sqrt{2\pi\sigma_k^2/2}} \int_{\sqrt{\frac{\gamma_l d^2}{2}}}^{\infty} \exp(-\frac{x^2}{2\sigma_k^2/2})\, dx, \tag{13}$$

which can be solved as

$$P_{PEP}^{(l)}(d|\gamma_l) = \sum_{k=0}^{C} \frac{e^{-A}A^k}{k!} Q(\sqrt{\frac{\gamma_l d^2}{\sigma_k^2}}), \tag{14}$$

where $Q(.)$ is the Gaussian Q-function [1, eq. (4.1)].

The unconditional pairwise error probability can be derived by averaging $P^{(l)}(d|\gamma_l)$ over the lth fading channel, i.e. $P^{(l)}(d) = \mathbb{E}_{\gamma_l}[P(d|\gamma_l)]$, yielding

$$P_{PEP}^{(l)}(d) = \sum_{k=0}^{C}\sum_{j=1}^{G} \frac{e^{-A}A^k\alpha_j}{k!\,\overline{\gamma}^{\beta_j}} \int_0^{\infty} x^{\beta_j-1} \times \exp(-\frac{\zeta_j x}{\overline{\gamma}}) Q(\sqrt{\frac{xd^2}{\sigma_k^2}})\, dx. \tag{15}$$

Using [30, eq. (11)] and [30, eq. (12)], we re-write (15) as

$$P_{PEP}^{(l)}(d) = \sum_{k=0}^{C}\sum_{j=1}^{G} \frac{\alpha_j}{2\sqrt{\pi}} \frac{e^{-A}A^k}{k!\overline{\gamma}^{\beta_j}} \int_0^{\infty} x^{\beta_j-1} \times G_{0,1}^{1,0}\left({}^{-,-}_{0,} \,|\, \frac{\zeta_j x}{\overline{\gamma}}\right) G_{1,2}^{2,0}\left(\begin{matrix} -,1 \\ 0,\frac{1}{2} \end{matrix} \middle| \frac{xd^2}{2\sigma_k^2} \right) dx, \tag{16}$$

where $G(.)$ is the Meijer's G-function [30, eq. (18)]. It is noted that (16) is the Mellin transformation of the product of the two Meijer's G-functions. Therefore, using [30, eq. (21)] and after some manipulations, we obtain

$$P_{PEP}^{(l)}(d) = \sum_{k=0}^{C}\sum_{j=1}^{G} \alpha_j \frac{e^{-A}A^k}{\sqrt{\pi}(k!)} (\frac{\sigma_k^2}{\overline{\gamma}d^2})^{\beta_j} \times 2^{2\beta_j-1} G_{2,2}^{1,2}\left({}^{1-\beta_j,\frac{1}{2}-\beta_j}_{0,-\beta_j} \,|\, \frac{2\zeta_j\sigma_k^2}{\overline{\gamma}d^2}\right), \tag{17}$$

where by using [30, eq. (17)], (17) can be re-written as

$$P_{PEP}^{(l)}(d) = \sum_{k=0}^{C}\sum_{j=1}^{G} \frac{\alpha_j A^k}{e^A k!} (\frac{\sigma_k^2}{2\overline{\gamma}d^2})^{\beta_j} \frac{\Gamma(2\beta_j)}{\Gamma(1+\beta_j)} \times {}_2\mathcal{F}_1(\beta_j, \frac{1}{2}+\beta_j, 1+\beta_j; \frac{-2\zeta_j\sigma_k^2}{d^2\overline{\gamma}}), \tag{18}$$

where ${}_2\mathcal{F}_1(.,.,.;.)$ is the Gauss hypergeometric function [23, eq. (9.113)].

For the MRC scheme, the received signal copies are coherently weighted and summed up in order to maximize the instantaneous output SNR, where the total instantaneous SNR at the output of the MRC method is given by

$$\gamma_\Sigma = \sum_{l=1}^{L} \gamma_l. \tag{19}$$

Generally, the PEP for the MRC scheme can be realized via the moment generating function (MGF) approach. By utilizing [1, eq. (4.2)] and [23, eq. (3.381.4)] and realizing that $M_{\gamma_\Sigma}(s) = \mathbb{E}_{\gamma_l}[\exp(\sum_{l=1}^{L}\gamma_l)] = \Pi_{l=1}^{L} M_{\gamma_l}(s)$, where $M_{\gamma_l}(s)$ is the MGF of the lth branch, one obtains the following single-fold integral

$$P_{PEP,\Sigma_L}(d) = \sum_{k=0}^{C} \frac{e^{-A}A^k}{k!\pi} \int_0^{\frac{\pi}{2}} \underbrace{[\sum_{j=1}^{K} \frac{\alpha_i\Gamma(\beta_j)}{(\frac{\overline{\gamma}d^2}{2\sigma_k^2\sin^2\phi}+\zeta_j)^{\beta_j}}]^L}_{M_{\gamma_\Sigma}(\frac{\overline{\gamma}d^2}{2\sigma_k^2\sin^2\phi})} d\phi. \tag{20}$$

However, in order to evaluate a rather more tractable PEP expression, one needs to derive the PDF of γ_Σ first. To this end, for the case of $L=2$, the PDF of γ_{Σ_2} can be obtained by

$$f_{\gamma_{\Sigma_2}}(\gamma) = \int_0^{\gamma} f_{\gamma_1}(x) f(\gamma - x) dx = \sum_{i,j=1}^{G}\sum_{j=1}^{G} \frac{\alpha_i}{\overline{\gamma}^{\beta_i}} \frac{\alpha_j}{\overline{\gamma}^{\beta_j}} \int_0^{\gamma} \frac{x^{\beta_i-1}(\gamma-x)^{\beta_j-1}}{e^{\frac{\zeta_i}{\overline{\gamma}}x} e^{\frac{\zeta_j}{\overline{\gamma}}(\gamma-x)}} dx. \tag{21}$$

In order to solve (21), we split the solution into two scenarios, namely when $\zeta_i = \zeta_j$ and $\zeta_i \neq \zeta_j$. In the former scenario, eq. (21) reduces to the following integral

$$f_{\gamma_{\Sigma_2}}|_{(\zeta_i=\zeta_j)} = \sum_{i=1}^{G}\sum_{j=1}^{G} \frac{\alpha_i}{\overline{\gamma}^{\beta_i}} \frac{\alpha_j}{\overline{\gamma}^{\beta_j}} e^{-\frac{\zeta_j}{\rho}\gamma} \int_0^{\gamma} x^{\beta_i-1}(\gamma-x)^{\beta_j-1} dx. \tag{22}$$

By the change of variables $u = \frac{x}{\gamma}$, and with the aid of [23, eq. (8.380)] and the functional relation in [23, eq. (8.384)], we obtain the following closed-form solution

$$f_{\gamma_{\Sigma_2}}|_{(\zeta_i=\zeta_j)} = \sum_{i=1}^{G}\sum_{j=1}^{G} \frac{\alpha_i\alpha_j}{\overline{\gamma}^{\beta_i+\beta_j}} \frac{\Gamma(\beta_i)\Gamma(\beta_j)}{\Gamma(\beta_i+\beta_j)} e^{-\frac{\zeta_j}{\rho}\gamma} \gamma^{\beta_i+\beta_j-1}. \tag{23}$$

Likewise, for the case $\zeta_i \neq \zeta_j$, eq. (22) is solved with the aid of the binomial theorem in [23, eq. (1.111)] and under the assumption that $\beta_j \in \mathbb{N}$. To this effect, the representation in (21) can be equivalently re-written as follows

$$f_{\gamma_{\Sigma_2}}|_{(\zeta_i\neq\zeta_j)} = \sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{l=0}^{\beta_j-1} \binom{\beta_j-1}{l} (-1)^l \frac{\alpha_i}{\overline{\gamma}^{\beta_i}} \frac{\alpha_j}{\overline{\gamma}^{\beta_j}} \gamma^{\beta_j-l-1} \times e^{-\frac{\zeta_j}{\overline{\gamma}}\gamma} \int_0^{\gamma} x^{\beta_i+l-1} e^{-\frac{x}{\overline{\gamma}}(\zeta_i-\zeta_j)} dx. \tag{24}$$

Evidently, the above integral can be expressed in closed-form with the aid of [23, eq. (8.350.1)] yielding

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$$f_{\gamma_{\Sigma_2}}|_{(\zeta_i \neq \zeta_j)} = \sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{l=0}^{\beta_j-1}\binom{\beta_j-1}{l}\frac{(-1)^l\alpha_i\alpha_j}{\overline{\gamma}^{\beta_j-l}(\zeta_i-\zeta_j)^{\beta_i+l}} \times \gamma^{\beta_j-l-1}e^{-\frac{\zeta_j}{\overline{\gamma}}\gamma}\tilde{\gamma}\left(\beta_i+l,\frac{\gamma(\zeta_i-\zeta_j)}{\overline{\gamma}}\right), \quad (25)$$

where $\tilde{\gamma}(a,x) \triangleq \int_0^x t^{a-1}e^{-t}dt$ denotes the lower incomplete gamma function. Thus, by expressing the $\tilde{\gamma}(a,x)$ function according to [23, eq. (8.352.6)] one obtains the following closed-form expression,

$$\begin{aligned} f_{\gamma_{\Sigma_2}}|_{(\zeta_i \neq \zeta_j)} &= \sum_{i=1}^{C}\sum_{j=1}^{C}\sum_{l=0}^{\beta_j-1}\binom{\beta_j-1}{l}\frac{(-1)^l\alpha_i\alpha_j}{\overline{\gamma}^{\beta_j-l}} \\ &\times (\zeta_i-\zeta_j)^{-\beta_i-l}\gamma^{\beta_j-l-1}e^{-\frac{\zeta_j}{\overline{\gamma}}\gamma}\Gamma(\beta_i+l) \\ &\times (1-e^{-\frac{\gamma(\zeta_i-\zeta_j)}{\overline{\gamma}}}\sum_{t=0}^{\beta_i+l-1}\frac{\left(\frac{\gamma(\zeta_i-\zeta_j)}{\overline{\gamma}}\right)^t}{t!}), \end{aligned} \quad (26)$$

which is valid for $\beta_i \in \mathbb{N}$, while

$$f_{\gamma_{\Sigma_2}} = f_{\gamma_{\Sigma_2}}|_{(\zeta_i=\zeta_j)} + f_{\gamma_{\Sigma_2}}|_{(\zeta_i \neq \zeta_j)}. \quad (27)$$

By following the same methodology, a similar expression can be obtained for $f_{\gamma_{\Sigma_3}}$ as was performed in our parallel work [31]. It is noted here that the above methodology allows for the derivation of similar expressions for $f_{\gamma_{\Sigma_4}}$, $f_{\gamma_{\Sigma_5}}$ and so forth.

Based on the above, the PEP for MRC is readily obtained by

$$P_{PEP,\Sigma_L}(d) = \sum_{k=0}^{C}\frac{e^{-A}A^k}{k!}\int_0^\infty f_{\gamma_\Sigma}^{(L)}(\gamma)\,Q(\sqrt{\frac{\gamma d^2}{2\sigma_k^2}})\,d\gamma. \quad (28)$$

For the case of $L=2$ and by inserting (23) and (26) in (28), it follows that

$$\begin{aligned} P_{PEP,\Sigma_2}(d)|_{(\zeta_i=\zeta_j)} &= \sum_{i,j=1}^{G}\sum_{k=0}^{C}\frac{\alpha_i\alpha_j}{\overline{\gamma}^{\beta_i+\beta_j}}\frac{\Gamma(\beta_i)\Gamma(\beta_j)}{\Gamma(\beta_i+\beta_j)}\frac{e^{-A}A^k}{k!} \\ &\times \int_0^\infty \frac{\gamma^{\beta_i+\beta_j-1}Q(\sqrt{\frac{\gamma d^2}{\sigma_k^2}})}{e^{-\frac{\zeta_j}{\overline{\gamma}}\gamma}}\,d\gamma, \end{aligned} \quad (29)$$

and

$$\begin{aligned} P_{PEP,\Sigma_2}(d)|_{(\zeta_i \neq \zeta_j)} &= \sum_{i,j=1}^{G}\sum_{k=0}^{C}\sum_{l=0}^{\beta_j-1}\binom{\beta_j-1}{l} \\ &\times \frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)}{\overline{\gamma}^{\beta_j-l}(\zeta_i-\zeta_j)^{\beta_i+l}}I_1(\overline{\gamma}) \\ &- \sum_{i,j=1}^{G}\sum_{k=0}^{C}\sum_{l=0}^{\beta_j-1}\sum_{t=0}^{\beta_i+l-1}\binom{\beta_j-1}{l}I_2(\overline{\gamma}) \\ &\times \frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)(\zeta_i-\zeta_j)^t}{\overline{\gamma}^{t+\beta_j-l}(\zeta_i-\zeta_j)^{\beta_i+l}t!}, \end{aligned} \quad (30)$$

where

$$I_1(\overline{\gamma}) = \int_0^\infty \gamma^{\beta_j-l-1}e^{-\frac{\zeta_j}{\overline{\gamma}}\gamma}Q(\sqrt{\frac{\gamma d^2}{\sigma_k^2}})d\gamma, \quad (31)$$

and

$$I_2(\overline{\gamma}) = \int_0^\infty \gamma^{t+\beta_j-l-1}e^{-\frac{\zeta_i}{\overline{\gamma}}\gamma}Q(\sqrt{\frac{\gamma d^2}{\sigma_k^2}})d\gamma. \quad (32)$$

Notably, the three integrals in (29) and (30) have the same algebraic representation as (15). Therefore, following a similar methodology presented in (16) and (17), and after some algebraic manipulations, yields (33) and (34) at the top of the next page. By following the methodology presented herein, the derivation of PEP expressions for higher diversity orders is readily available.

2) Pairwise Error Probability for Selection Combining: The equivalent SNR for the SC scheme is given by

$$\gamma_{SC} = \max_{l=\{1,\ldots,L\}} \gamma_l. \quad (35)$$

Accordingly, we obtain $F_{\gamma_{SC}}(x) = \Pi_{l=1}^{L}F_{\gamma_l}(x)$, where

$$F_{\gamma_1}(x) = \sum_{i=1}^{G}\alpha_i\zeta_i^{-\beta_i}\tilde{\gamma}(\beta_i,\frac{\zeta_i x}{\overline{\gamma}}) \quad (36)$$

is the cumulative distribution function (CDF) of γ_l. For the case of two independent and identically distributed (*i.i.d.*) branches, differentiating $F_{\gamma_{SC}}(x)$ with respect to x yields

$$f_{\gamma_{SC_2}}(x) = \sum_{i=1}^{G}\sum_{j=1}^{G}\frac{2\alpha_i\alpha_j}{\overline{\gamma}\zeta_j^{\beta_j}}(\frac{x}{\overline{\gamma}})^{\beta_i-1}\exp(\frac{-\zeta_i x}{\overline{\gamma}})\tilde{\gamma}(\beta_j,\frac{\zeta_j x}{\overline{\gamma}}). \quad (37)$$

Similar to (20), generally a corresponding PEP expression for the SC scheme can be similarly expressed via the MGF approach, where $M_{\gamma_{sc}}(s)$ is derived using [23, eq. (6.45.2)], yielding

$$\begin{aligned} P_{PEP,SC_2}(d) &= \sum_{k=0}^{C}\frac{2A^k e^{-A}}{\pi(k!)}\int_0^{\frac{\pi}{2}}\overbrace{\sum_{i,j=1}^{G}\frac{\alpha_i\alpha_j\Gamma(\beta_{ji})}{\beta_i(\zeta_{ji}-{}^{s}/_{\sin^2\theta})^{\beta_{ji}}}}^{M_{\gamma_{SC}}({}^{s}/_{\sin^2\theta})} \\ &\times \underbrace{{}_2\mathcal{F}_1(1,\beta_{ji};\beta_i+1;\frac{\zeta_i}{(\zeta_{ji}-{}^{s}/_{\sin^2\theta})}}\,)d\theta, \end{aligned} \quad (38)$$

where $s=\frac{\overline{\gamma}d^2}{2\sigma_k^2}$, $\beta_{ji}=\beta_j+\beta_i$, and $\zeta_{ji}=\zeta_j+\zeta_i$.

A rather more tractable expression valid for $\beta_j \in \mathbb{N}$ can be obtained as follows. By expressing $\tilde{\gamma}(\beta_j,\frac{\zeta_j x}{\overline{\gamma}})$ according to [23, eq. (8.352.6)], eq. (37) can be re-written as

$$\begin{aligned} f_{\gamma_{SC_2}}(x) &= \sum_{i=1}^{G}\sum_{j=1}^{G}\frac{2\alpha_i\alpha_j}{\overline{\gamma}\zeta_j^{\beta_j}}(\frac{x}{\overline{\gamma}})^{\beta_i-1}\exp(\frac{-\zeta_i x}{\overline{\gamma}}) \\ &\times \Gamma(\beta_j)\Big(1-e^{-\frac{\zeta_j x}{\overline{\gamma}}}\sum_{t=0}^{\beta_j-1}\frac{(\frac{\zeta_j x}{\overline{\gamma}})^t}{t!}\Big). \end{aligned} \quad (39)$$

Based on the above PDF, the PEP for SC can be obtained by (28) with replacing γ_Σ with γ_{SC}. Similarly, eq. (39) has the same algebraic representation as in (15). Therefore, by following the method presented in (17) and (18) and after some algebraic manipulations, we obtain (40) at the top of the next page.

3) Symbol Error Probability and Bit Error Probability: Having the PEP expressions derived, at large $\overline{\gamma}$, the SEP of

$$P_{PEP,\Sigma_2}(d)|_{(\zeta_i=\zeta_j)} = \sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{k=0}^{C} \frac{\alpha_i\alpha_j\Gamma(\beta_i)\Gamma(\beta_j)}{\Gamma(\beta_i+\beta_j)} \frac{\Gamma(2\beta_j+2\beta_i)}{\Gamma(1+\beta_j+\beta_i)} \frac{e^{-A}A^k}{k!} \times (\frac{\sigma_k^2}{2\overline{\gamma}d^2})^{\beta_i+\beta_j} {}_2\mathcal{F}_1(\beta_i+\beta_j, \frac{1}{2}+\beta_i+\beta_j, 1+\beta_i+\beta_j; -\frac{2\zeta_j\sigma_k^2}{d^2\overline{\gamma}}), \quad (33)$$

$$P_{PEP,\Sigma_2}(d)|_{(\zeta_i\neq\zeta_j)} = \sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{k=0}^{C}\sum_{l=0}^{\beta_j-1} \binom{\beta_j-1}{l} \frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)}{(\zeta_i-\zeta_j)^{\beta_i+l}} \frac{\Gamma(2(\beta_j-l))}{\Gamma(1+\beta_j-l)} \times (\frac{\sigma_k^2}{2\overline{\gamma}d^2})^{\beta_j-l} {}_2\mathcal{F}_1(\beta_j-l, \frac{1}{2}+\beta_j-l, 1+\beta_j-l; -\frac{2\zeta_j\sigma_k^2}{d^2\overline{\gamma}}) - \sum_{i=1}^{G}\sum_{j=1}^{G} \times \sum_{k=0}^{C}\sum_{l=0}^{\beta_j-1}\sum_{t=0}^{\beta_i+l-1} \binom{\beta_j-1}{l} \frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)}{(\zeta_i-\zeta_j)^{\beta_i+l-t}t!} \frac{\Gamma(2(t+\beta_j-l))}{\Gamma(1+t+\beta_j-l)} \times (\frac{\sigma_k^2}{2\overline{\gamma}d^2})^{t+\beta_j-l} {}_2\mathcal{F}_1(t+\beta_j-l, \frac{1}{2}+t+\beta_j-l, 1+t+\beta_j-l; -\frac{2\zeta_i\sigma_k^2}{d^2\overline{\gamma}}). \quad (34)$$

$$P_{PEP,SC_2}(d) = \sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{k=0}^{C} \frac{2e^{-A}A^k}{k!} \frac{\alpha_i\alpha_j\Gamma(\beta_j)}{\zeta_j^{\beta_j}} \frac{\Gamma(2\beta_i)}{\Gamma(1+\beta_i)} (\frac{\sigma_k^2}{2\overline{\gamma}d^2})^{\beta_i} \times {}_2\mathcal{F}_1(\beta_i, \frac{1}{2}+\beta_i, 1+\beta_i; \frac{-2\zeta_i\sigma_k^2}{d^2\overline{\gamma}}) - \sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{k=0}^{C}\sum_{t=0}^{\beta_j-1} \frac{2e^{-A}A^k}{k!} \frac{\alpha_i\alpha_j\Gamma(\beta_j)}{\zeta_j^{\beta_j-t}t!} \times \frac{\Gamma(2(t+\beta_i))}{\Gamma(1+t+\beta_i)} (\frac{\sigma_k^2}{2\overline{\gamma}d^2})^{t+\beta_i} {}_2\mathcal{F}_1(t+\beta_i, \frac{1}{2}+t+\beta_i, 1+t+\beta_i; \frac{-2(\zeta_i+\zeta_j)\sigma_k^2}{d^2\overline{\gamma}}). \quad (40)$$

various M-ary linear signaling schemes can be expressed as [16]

$$P_s = 2\eta_M P(d_M), \quad (41)$$

where d_M is the minimum average Euclidean distance of $\mathcal{S}$, and η_M is a parameter dependent on the signaling scheme, as summarized in [16, Table. 1]. Assuming gray coding, the BEP can be determined by $P_b = {}^{P_s}/_{\log_2 M}$,where M is the size of the constellation $\mathcal{S}$. For the case of binary phase shift keying (BPSK), the exact SEP is obtained when $\eta_M = \frac{1}{2}$, and $\mathrm{d}_M = \sqrt{2}$. The BPSK signaling is considered in our simulations and analyses thereafter.

C. Average Channel Capacity

Assuming that the channel state information is known, the conditional channel capacity of a two-term mixture distribution, which can be written as

$$f_N(x) = p_0 f_B(x) + (1-p_0) f_I(x), \quad (42)$$

where $0 < p_0 \leq 1$ is the mixture weight, and $f_B(x)$, $f_I(x)$ are the individual densities, was derived as [32]

$$\mathcal{C} = p_0\mathcal{C}^{(0)} + p_1\mathcal{C}^{(1)}, \quad (43)$$

where $\mathcal{C}^{(0)}$ and $\mathcal{C}^{(1)}$ are the capacities of $f_B(x)$ and $f_I(x)$, respectively. Extending on this approach, we can express the average unconditional capacity under the MCA noise as a weighted summation of the individual capacities of each Gaussian component as

$$\overline{\mathcal{C}} = \sum_{k=0}^{C} \frac{e^{-A}A^k B}{k!\ln 2} \mathbb{E}_\gamma\{\ln(1+\frac{\overline{\gamma}}{\sigma_k^2}\gamma)\}. \quad (44)$$

Denote the effective average SNR per kth noise component as $\overline{\gamma}_k = \frac{\overline{\gamma}}{\sigma_k^2}$, then following [10], the average channel capacity for the single-branch scenario, which is valid for $\beta_i \in \mathbb{N}$, can be expressed in closed-form as

$$\overline{\mathcal{C}} = \sum_{k=0}^{C}\sum_{i=1}^{G}\sum_{k=1}^{\beta_i} \alpha_i \frac{e^{-A}A^k}{k!\,\overline{\gamma}_k^{\beta_i}} \frac{B}{\ln 2} \Gamma(\beta_i) e^{\frac{\zeta_i}{\overline{\gamma}_k}} \frac{\Gamma(k-\beta_i, \frac{\zeta_i}{\overline{\gamma}_k})}{(\frac{\zeta_i}{\overline{\gamma}_k})^k}. \quad (45)$$

For the MRC scheme, since $f_{\gamma_{MRC}}(x)$ has the same algebraic representation as the MG distribution, the average channel capacity for $L = 2$ is readily obtained in (46) and (47) at the top of the next page, while

$$\overline{\mathcal{C}}_{\Sigma_2} = \overline{\mathcal{C}}_{\Sigma_2}|_{(\zeta_i=\zeta_j)} + \overline{\mathcal{C}}_{\Sigma_2}|_{(\zeta_i\neq\zeta_j)}. \quad (48)$$

Similar expressions for the average channel capacity for higher diversity orders can be obtained by following the same methodology. Likewise, for the SC scheme, an expression for the average channel capacity for $L = 2$ can be derived from (39) as in (49) at the top of the next page, which is valid for $\beta_i, \beta_j \in \mathbb{N}$. In order to get expressions valid for all values

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2016.2569064, IEEE Transactions on Vehicular Technology

$$\overline{\mathcal{C}}_{\Sigma_2}|_{(\zeta_i=\zeta_j)} = \sum_{k=0}^{C}\sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{k=1}^{\beta_i+\beta_j} \frac{e^{-A}A^k\alpha_i\alpha_j}{k!\,\overline{\gamma}_k^{\beta_i+\beta_j}}\frac{B}{\ln 2}\Gamma(\beta_i)\Gamma(\beta_j)\, e^{\frac{\zeta_j}{\overline{\gamma}_k}}\frac{\Gamma(k-\beta_i+\beta_j,\frac{\zeta_j}{\overline{\gamma}_k})}{(\frac{\zeta_j}{\overline{\gamma}_k})^k}, \tag{46}$$

$$\begin{aligned}\overline{\mathcal{C}}_{\Sigma_2}|_{(\zeta_i\neq\zeta_j)} &= \sum_{k=0}^{C}\sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{l=0}^{\beta_j-1}\sum_{k=1}^{\beta_j-l}\frac{B}{\ln 2}\binom{\beta_j-1}{l}\frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)}{\overline{\gamma}_k^{\beta_j-l}(\zeta_i-\zeta_j)^{\beta_i+l}}\Gamma[\beta_j-l]\, e^{\frac{\zeta_j}{\overline{\gamma}_k}}\frac{\Gamma(k-\beta_j-l,\frac{\zeta_j}{\overline{\gamma}_k})}{(\frac{\zeta_j}{\overline{\gamma}_k})^k}\\ &- \sum_{k=0}^{C}\sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{l=0}^{\beta_j-1}\sum_{t=0}^{\beta_i+l-1}\sum_{k=1}^{\beta_j+t-l}\frac{e^{-A}A^k}{k!}\frac{B}{\ln 2}\binom{\beta_j-1}{l}\\ &\times \frac{(-1)^l\alpha_i\alpha_j\Gamma(\beta_i+l)}{\overline{\gamma}_k^{\beta_j+t-l}(\zeta_i-\zeta_j)^{\beta_i-t+l}t!}\Gamma[\beta_j+t-l]\, e^{\frac{\zeta_i}{\overline{\gamma}\, k}}\frac{\Gamma(k-\beta_j+t-l,\frac{\zeta_i}{\overline{\gamma}_k})}{(\frac{\zeta_i}{\overline{\gamma}_k})^k},\end{aligned} \tag{47}$$

$$\begin{aligned}\overline{\mathcal{C}}_{SC_2} &= \sum_{k=0}^{C}\sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{k=1}^{\beta_i}\frac{e^{-A}A^k}{k!}\frac{B}{\ln 2}\frac{2\alpha_i\alpha_j\Gamma(\beta_j)}{\overline{\gamma}_k^{\beta_i}\zeta_j^{\beta_j}}(\beta_i-1)!\, e^{\frac{\zeta_i}{\overline{\gamma}_k}}\frac{\Gamma(k-\beta_i,\frac{\zeta_i}{\overline{\gamma}_k})}{(\frac{\zeta_i}{\overline{\gamma}_k})^k}\\ &- \sum_{k=0}^{C}\sum_{i=1}^{G}\sum_{j=1}^{G}\sum_{t=0}^{\beta_j-1}\frac{e^{-A}A^k}{k!}\frac{B}{\ln 2}\frac{2\alpha_i\alpha_j\Gamma(\beta_j)}{\overline{\gamma}_k^{\beta_i+t}\zeta_j^{\beta_j-t}t!}(t+\beta_i-1)!\, e^{\frac{\zeta_i+\zeta_j}{\overline{\gamma}_k}}\sum_{k=1}^{t+\beta_i}\frac{\Gamma(k-t+\beta_i,\frac{\zeta_i+\zeta_j}{\overline{\gamma}_k})}{(\frac{\zeta_i+\zeta_j}{\overline{\gamma}_k})^k},\end{aligned} \tag{49}$$

of the involved parameter β, we obtain tight approximate expressions for the average channel capacity as follows. By utilizing the Taylor's series, $\ln(1+\overline{\gamma}_k\gamma)\}$ can be expanded about $\overline{\gamma}_k$, yielding

$$\ln(1+\overline{\gamma}_k\gamma) = \ln(1+\mathbb{E}[\frac{\gamma}{\sigma_i^2}]) + \sum_{w=1}^{\infty}\frac{(-1)^{w-1}}{w}\frac{(x-\mathbb{E}[\frac{\gamma}{\sigma_k^2}])^w}{(1+\mathbb{E}[\frac{\gamma}{\sigma_k^2}])^w}. \tag{50}$$

By taking the expectation of this expansion and truncating the result to contain the first two moments, i.e. $w=2$, we obtain an approximate expression

$$\overline{\mathcal{C}} \approx \sum_{k=0}^{C}\frac{e^{-A}A^kB}{k!\pi\ln2}[\ln(1+\mathbb{E}[\frac{\gamma}{\sigma_k^2}]) - \frac{\mathbb{E}[(\frac{\gamma}{\sigma_k^2})^2]-\mathbb{E}^2[\frac{\gamma}{\sigma_k^2}]}{2(1+\mathbb{E}[\frac{\gamma}{\sigma_k^2}])^2}]. \tag{51}$$

Similarly, for the case of ϵ-mixture noise, the capacity can be expressed as

$$\begin{aligned}\overline{\mathcal{C}} &\approx \frac{B}{\ln2}\Big((1-\epsilon)[\ln(1+\mathbb{E}[\frac{\gamma}{\sigma_g^2}]) - \frac{\mathbb{E}[(\frac{\gamma}{\sigma_g^2})^2]-\mathbb{E}^2[\frac{\gamma}{\sigma_g^2}]}{2(1+\mathbb{E}[\frac{\gamma}{\sigma_g^2}])^2}]\\ &+ \epsilon[\ln(1+\mathbb{E}[\frac{\gamma}{\sigma_i^2}]) - \frac{\mathbb{E}[(\frac{\gamma}{\sigma_i^2})^2]-\mathbb{E}^2[\frac{\gamma}{\sigma_i^2}]}{2(1+\mathbb{E}[\frac{\gamma}{\sigma_i^2}])^2}]\Big).\end{aligned} \tag{52}$$

Here we derive the first two moments $\mathbb{E}[\frac{\gamma}{\sigma_k^2}]$ and $\mathbb{E}[(\frac{\gamma}{\sigma_k^2})^2]$ by differentiating the corresponding MGF as follows,

$$\mathbb{E}[(\frac{\gamma}{\sigma_k^2})^n] = \frac{d^{(n)}M_\gamma(\frac{s}{\sigma_k^2})}{ds^{(n)}}\,|_{s=0}\,. \tag{53}$$

Differentiating $M_{\gamma_{MRC}}(s)$ in (20) using (53), and after some mathematical simplifications, we obtain

$$\mathbb{E}[\frac{\gamma_{MRC}}{\sigma_k^2}] = -L(\frac{\overline{\gamma}_{MRC}}{\sigma_k^2})\Lambda^{L-1}\Omega, \tag{54}$$

$$\mathbb{E}[(\frac{\gamma_{MRC}}{\sigma_k^2})^2] = L(\frac{\overline{\gamma}_{MRC}}{\sigma_k^2})^2\Big[(L-1)\Lambda^{L-2}\Omega^2+\Lambda^{L-1}\Theta\Big], \tag{55}$$

where $\Lambda = \sum_{i=1}^{K}\frac{\alpha_i\Gamma(\beta_i)}{\zeta_i^{\beta_i}}, \Omega = \sum_{j=1}^{K}\frac{\beta_j\alpha_j\Gamma(\beta_j)}{\zeta_j^{\beta_j+1}}$, and $\Theta = \sum_{j=1}^{K}\frac{\beta_j\alpha_j\Gamma(\beta_j)(\beta_j+1)}{\zeta_j^{\beta_j+2}}$.

Similarly, differentiating $M_{\gamma_{SC}}(s)$ in (38) using (53), and after considerable mathematical manipulation, we arrive at

$$\mathbb{E}[\frac{\gamma_{SC}}{\sigma_k^2}] = \sum_{i,j=1}^{G}\Psi_1(i,j)\Big[\Phi_1(i,j)+\frac{\zeta_j\Phi_2(i,j)}{\zeta_{ij}(\beta_j+1)}\Big], \tag{56}$$

$$\begin{aligned}\mathbb{E}[(\frac{\gamma_{SC}}{\sigma_k^2})^2] &= \sum_{i,j=1}^{G}\frac{(\beta_{ij}+1)\Psi_2(i,j)}{\zeta_{ij}}\Big[\Phi_1(i,j)\\ &+ \frac{2\zeta_j\Phi_2(i,j)}{\zeta_{ij}(\beta_j+1)}+\frac{2\zeta_j^2\Phi_3(i,j)}{(\beta_j+1)\zeta_{ij}^2(\beta_j+2)}\Big],\end{aligned} \tag{57}$$

where $\Psi_y(i,j) = (\frac{\overline{\gamma}_{SC}}{\sigma_k^2})^y\frac{2\beta_{ij}\alpha_i\alpha_j\Gamma(\beta_{ij})}{\beta_i\zeta_{ij}^{\beta_{ij}+1}}$, $\Phi_y(i,j) = {}_2\mathcal{F}_1\Big(y,\beta_{ij}+y-1;\beta_j+y;\frac{\zeta_j}{\zeta_{ij}}\Big)$.

V. Simulations and Discussion

In order to evaluate the effectiveness of the BIC described in Section III and to validate the derivations of Section IV-A, Fig. 2 depicts the analytical and simulated SEP for BPSK signaling under MCA noise with $\Xi = 0.01$, $A = 0.1$ for both MRC and SC schemes with $L = 4, \text{and}\, 2$, respectively, for various MG-based fading channels, namely NL, K_G, $\kappa-\mu$, and $\eta-\mu$. The number of components is chosen according to the BIC, as indicated in Section III. As shown, the analytical SEP curves are very accurate over the whole operating average SNR

This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2016.2569064, IEEE Transactions on Vehicular Technology

region for both MRC and SC. This finding directly reflects the accuracy and effectiveness of the proposed BIC approach to select the number of the components. In the simulations, the term Monte Carlo indicates that we utilized the actual fading channel variates, with a number of repetitions of 10^6 trials or bits.

In Fig. 3, we analyze the SEP for the MRC scheme, when $L = 1$ and $L = 4$, with the fading channel following the NL fading model. We consider several scenarios of the MCA noise with $\Xi = 0.1$, and $A = (0.1, 0.3, 0.9)$. Recall that as $A \to 0$, the noise becomes more impulsive. Therefore, as one would expect, at medium and large $\overline{\gamma}$, the SEP curve becomes more flat as A tends to zero. However, interestingly we notice that this is not the case at very small $\overline{\gamma}$, where there exists a threshold, marked by arrows, for which the error performance associated with small A (more impulsive scenario) is better. Increasing the diversity order shifts this threshold to the left; when $L = 1$, the threshold was at $\overline{\gamma} = 5$ dB, whereas with $L = 4$, the threshold shifts to $\overline{\gamma} = 0$ dB.

Finally, in Fig. 4, we plot the average channel capacity versus ϵ for the NL and $\eta - \mu$ fading scenarios assuming MRC diversity scheme and ϵ-mixture noise with an impulsive index of $\xi = \sigma_i^2/\sigma_g^2 = 75$. First, we notice that as the diversity order increases, the whole capacity curve shifts upwards. Furthermore, when ϵ is either small or large, one Gaussian component dominates, resulting in low average channel capacity merit. Here the skewness of the curve reflects the fact that at large ϵ, the noise becomes more impulsive.

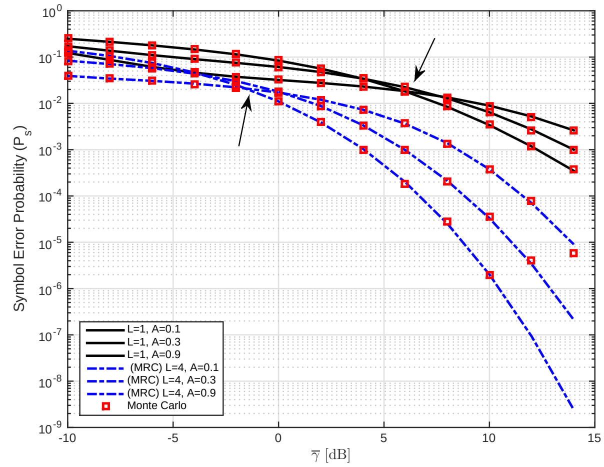

Figure 3. Analytical and simulated SEP of BPSK with MRC scheme for NL fading contaminated with MCA Noise of $\lambda = 0.1$, $A = (0.1, 0.3, 0.9)$, and $C = 10$.

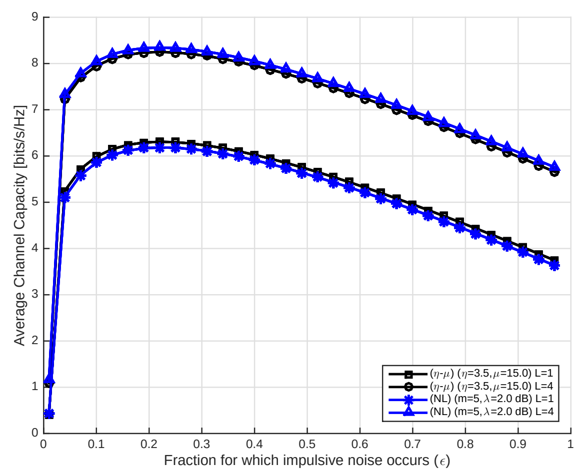

Figure 4. Average capacity with and without MRC diversity for NL and $\eta - \mu$ fading channels contaminated with ϵ-mixture noise with $\overline{\gamma} = 10$ dB and $\xi = 75$.

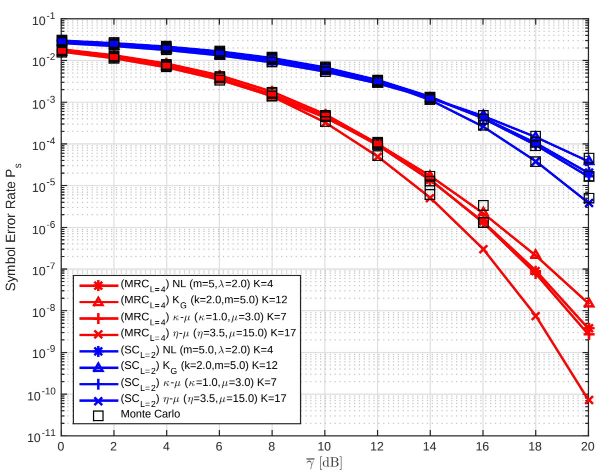

Figure 2. Analytical and simulated SEP of BPSK with 4-MRC and 2-SC schemes for various MG based fading channels with MCA Noise of $\Xi = 0.01$, $A = 0.1$, and $C = 10$.

VI. Conclusions

In this paper, we have proposed a unified and versatile approach to the performance analysis of SIMO over generalized and composite fading channels with impulsive noise. Specifically, we have proposed an effective-information theoretic approach to determine the optimal number of components for various MG-based generalized and composite fading models, based on the BIC. We also have derived novel exact analytical PEP and average channel capacity expressions for the performance of SIMO systems with MRC and SC schemes over the MG-based fading channels with impulsive noise, modeled by Middleton's Class-A (MCA) and ϵ-mixture noise models. Our derived expressions have been shown to be both algebraically versatile and generalized to many fading channels and noise environments.

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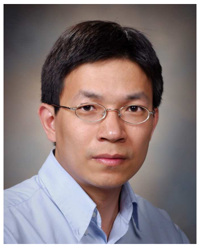
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TVT.2016.2569064, IEEE Transactions on Vehicular Technology



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